

Metropolis Monte Carlo
sampling

Gibbs Clustering

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Metropolis Monte Carlo

- What to do if you cannot do the math or your error function cannot be differentiated?

$$\frac{\partial E}{\partial w_i} = ?$$

Example: Estimating π by Independent Monte-Carlo Samples

Suppose we throw darts randomly (and uniformly) at the square:

$$\frac{\# \text{ darts hitting shaded area}}{\# \text{ darts hitting inside square}} = \frac{\frac{1}{4} \pi r^2}{r^2} = \frac{1}{4} \pi$$

or

$$\pi = 4 \frac{\# \text{ darts hitting shaded area}}{\# \text{ darts hitting inside square}}$$

Algorithm:

For $i=[1..n\text{trials}]$

$x = (\text{random\# in } [0..r])$

$y = (\text{random\# in } [0..r])$

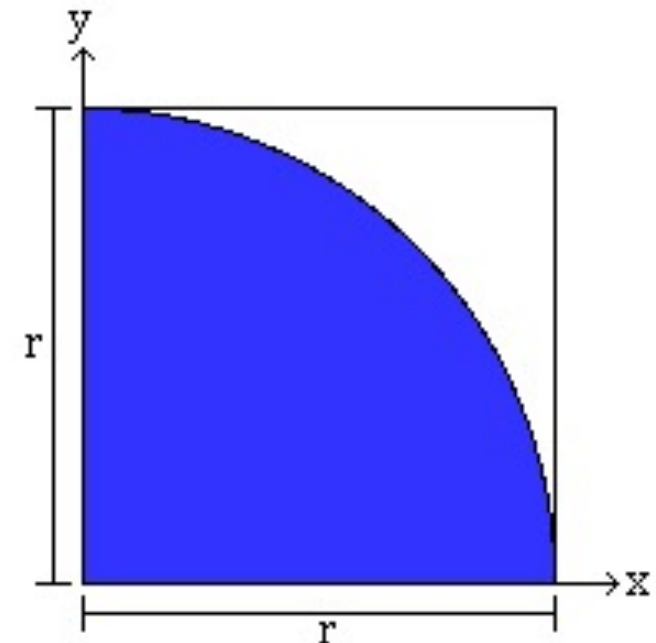
$\text{distance} = \text{sqrt}(x^2 + y^2)$

if $\text{distance} \leq r$

$\text{hits}++$ $4 \times \text{hits}$

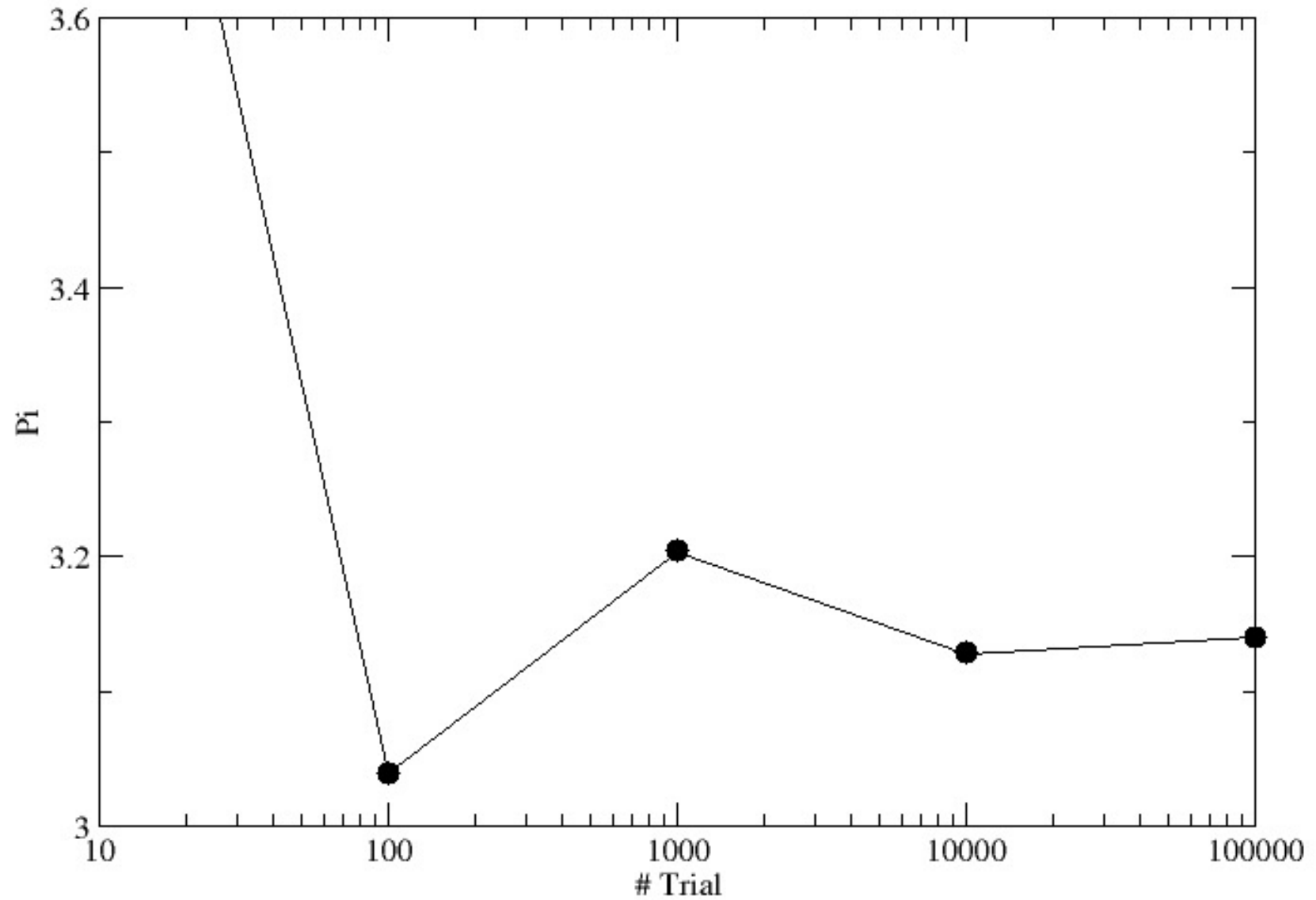
End

Output: $\frac{\text{hits}}{n\text{trials}}$



<http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html>

Estimating Π



Monte Carlo

Because of their reliance on repeated computation of random or pseudo-random numbers, Monte Carlo methods are most suited to calculation by a computer. Monte Carlo methods tend to be used when it is unfeasible or impossible to compute an exact result with a deterministic algorithm

Or when you are too stupid to do the math yourself?

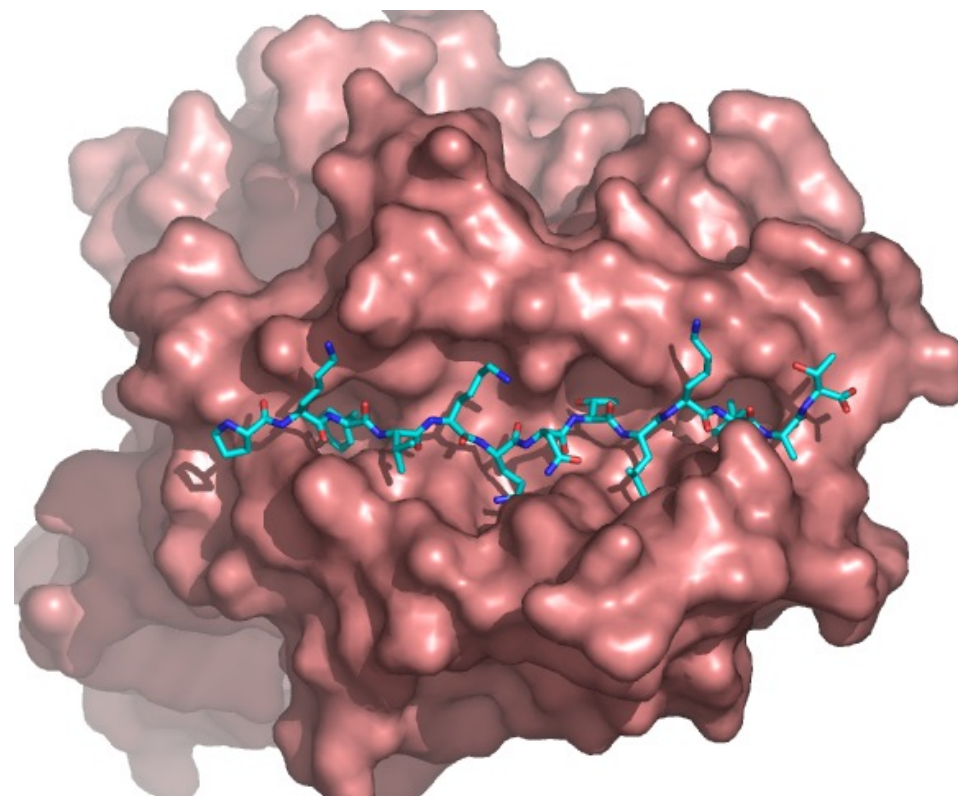
$$E = f(x)$$

$$dE = E_1 - E_0$$

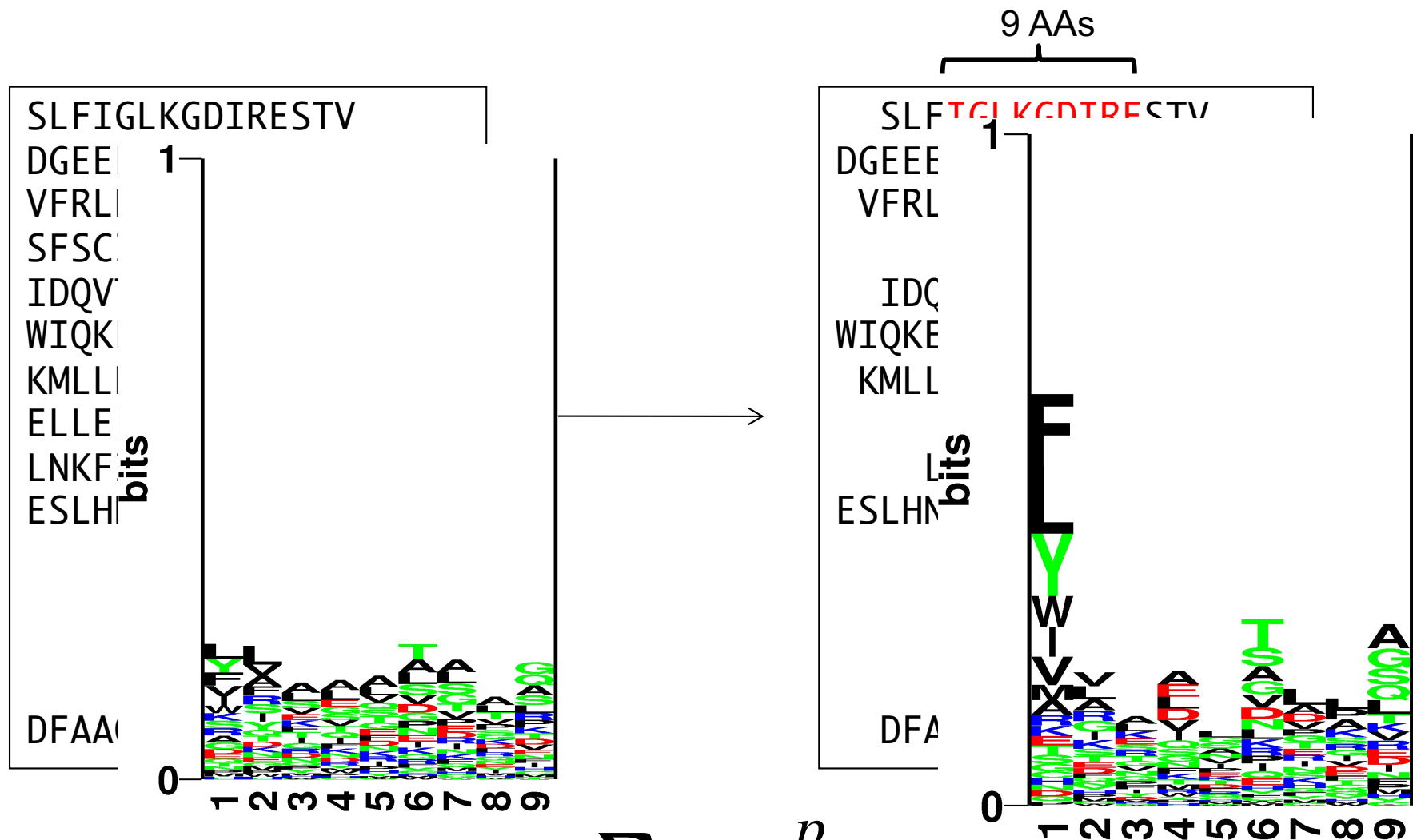
$$P(\text{accept}) = \min(1, e^{-dE/T})$$

Class II MHC binding

- MHC class II binds peptides in the class II antigen presentation pathway
- Binds peptides of length 9-18 (even whole proteins can bind!)
- Binding cleft is open
- Binding core is 9 aa



Conventional Gibbs sampling MHC class II binding



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$$E = \sum_{\text{peptides}} \log \frac{P_{p,a}}{q_a}$$

Gibbs sampling - sequence alignment

Why sampling?

50 sequences 12 amino acids long

try all possible combinations with a 9-mer overlap



$4^{50} \sim 10^{30}$ possible combinations

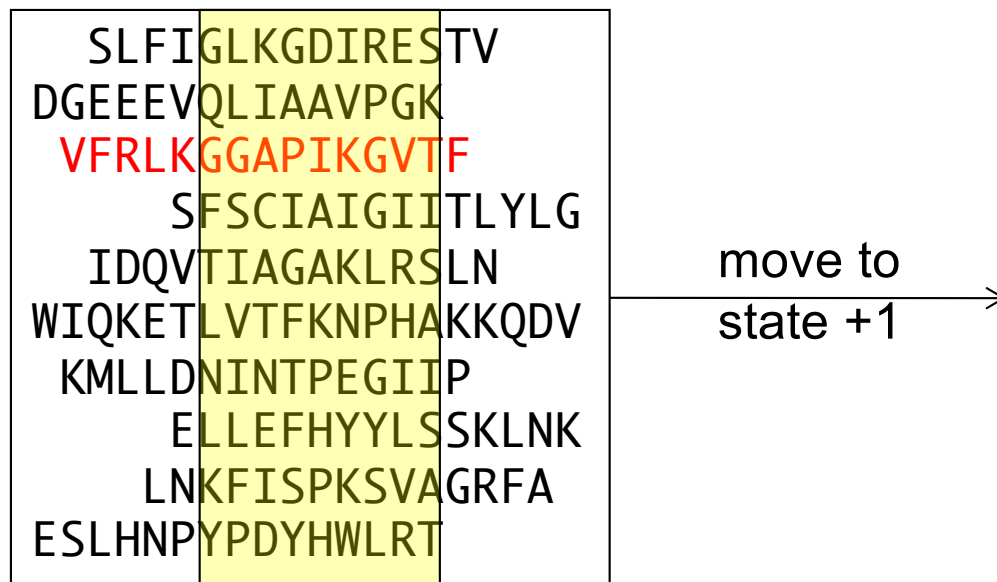
...computationally unfeasible

```

SLFIGLKGDIRESTV
DGEEEVQLIAAVPGK
VFRLKGGAPIKGVTF
      SFSCIAIGIITLYLG
IDQVTIAGAKLRSLN
WIQKETLVTFKNPHAKKQDV
KMLLDNINTPEGIIP
      ELLEFHYYLSSKLNK
      LNKFISPKSVAGRFA
ESLHNPYPDYHWLRT
      ...
      ...
      ...
      ...
DFAAQVDYPSTGLY
  
```


Gibbs sampling - sequence alignment

State transition

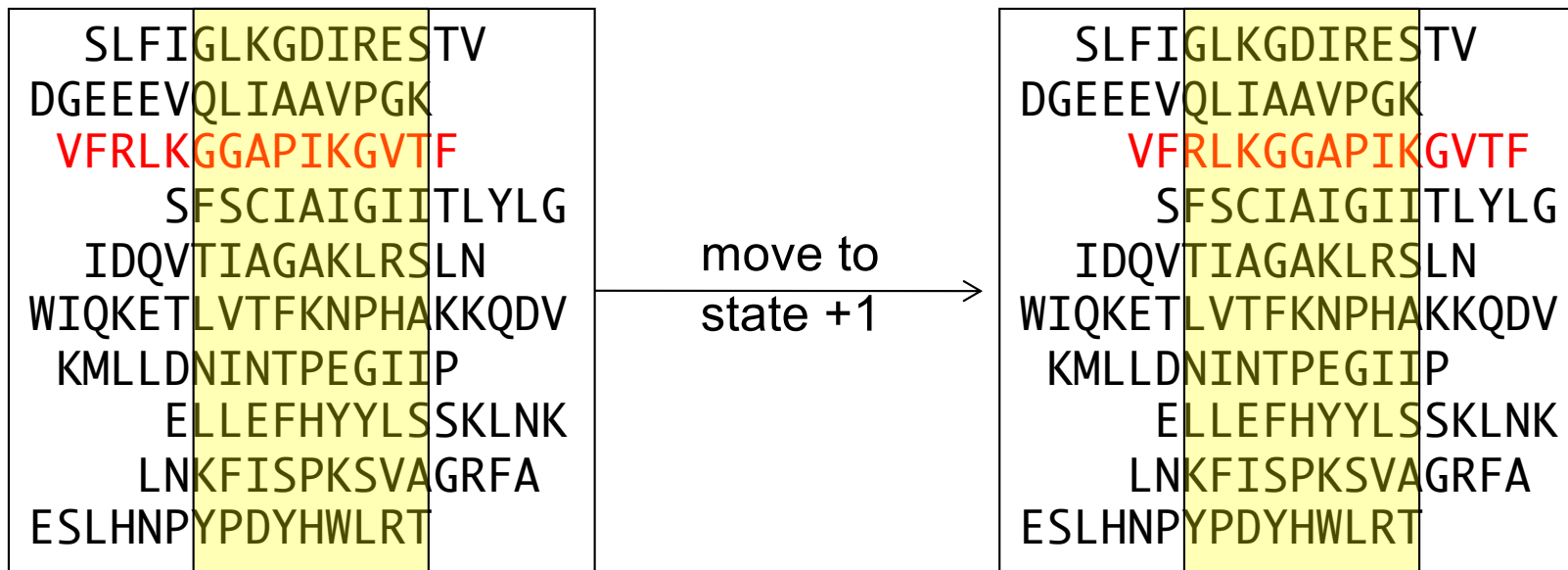


$$E = \sum_{\text{peptides}} \log \frac{P_{p,a}}{q_a}$$

$$dE = E_i - E_{i-1}$$

Gibbs sampling - sequence alignment

State transition



$$E = \sum_{\text{peptides}} \log \frac{p_{p,a}}{q_a}$$

$$dE = E_i - E_{i-1}$$

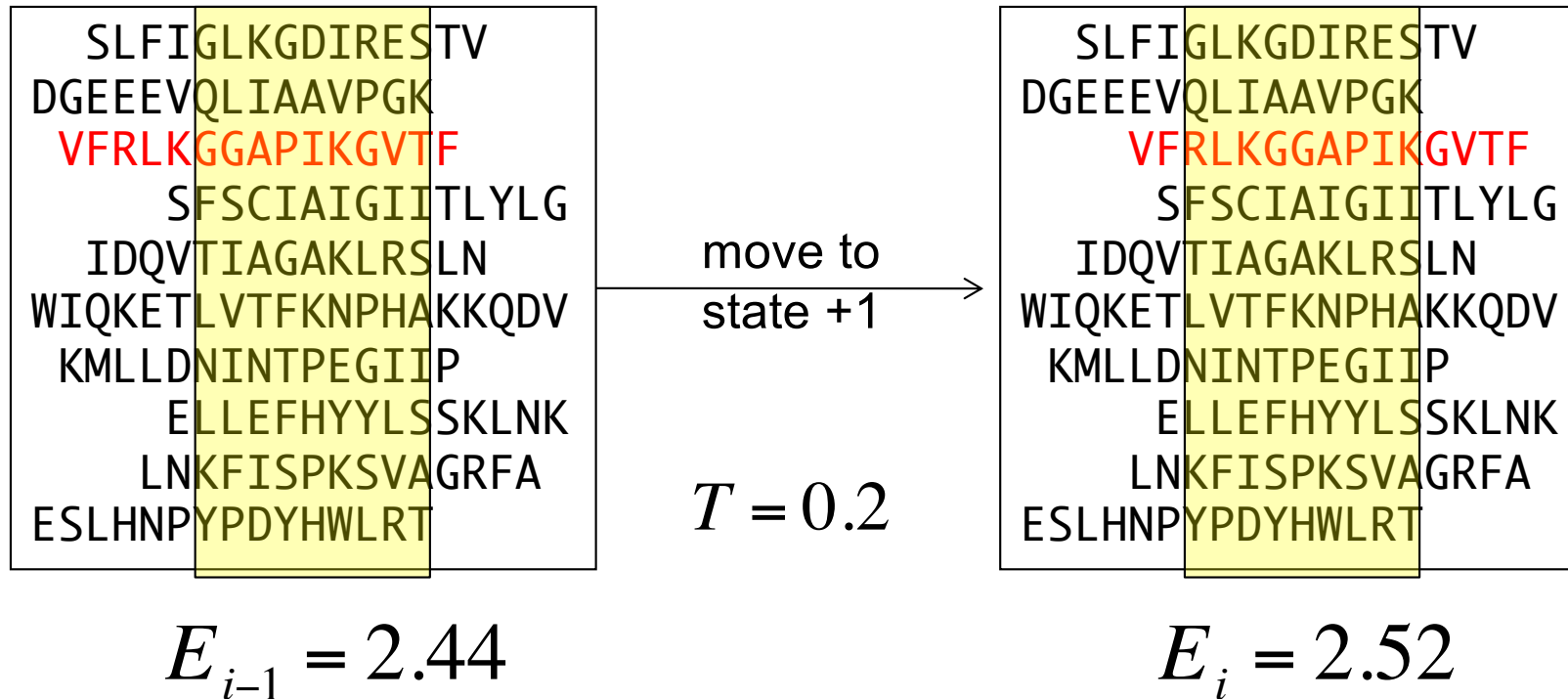
Accept or reject the move?

$$P = \min \left[1, \exp \left(\frac{dE}{T} \right) \right]$$

Note that the probability of going to the new state depends on the previous state only

Gibbs sampling - sequence alignment

Numerical example - 1

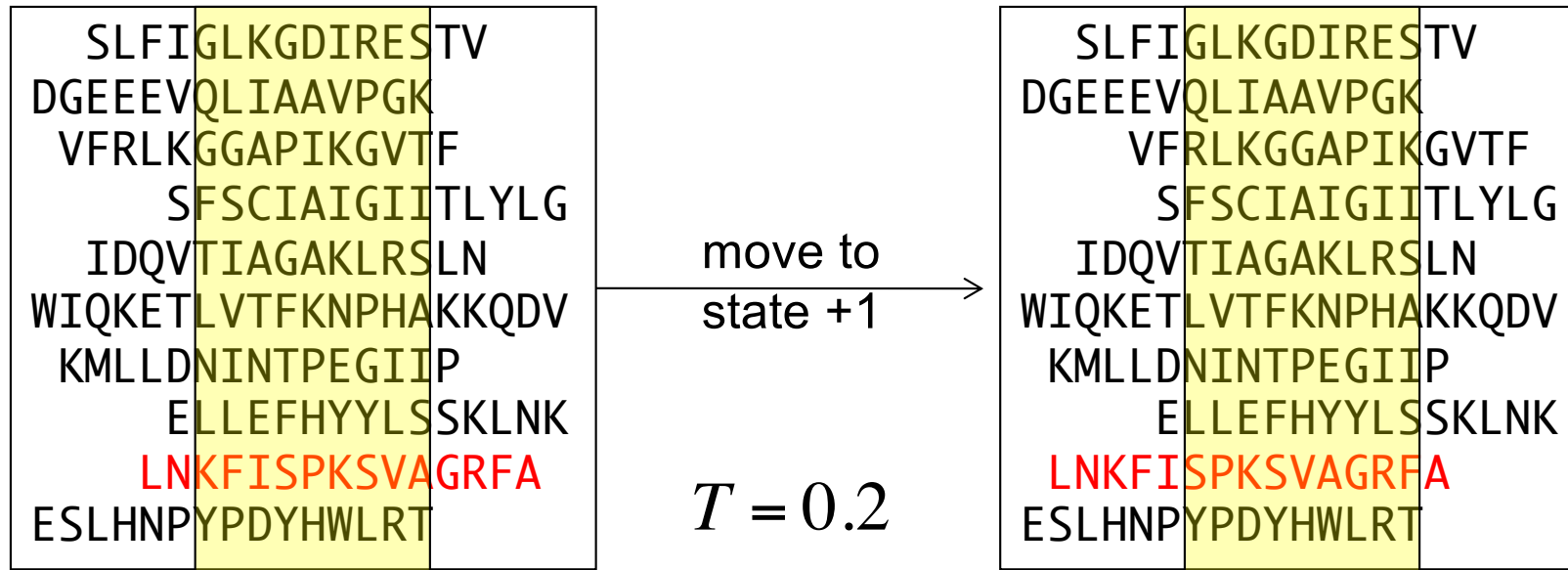


$$P = \min\left[1, \exp\left(\frac{0.08}{0.2}\right)\right] = \min[1, 1.49] = 1$$

**Accept move with
 Prob = 100%**

Gibbs sampling - sequence alignment

Numerical example - 2



$$E_{i-1} = 2.44$$

$$E_i = 2.35$$

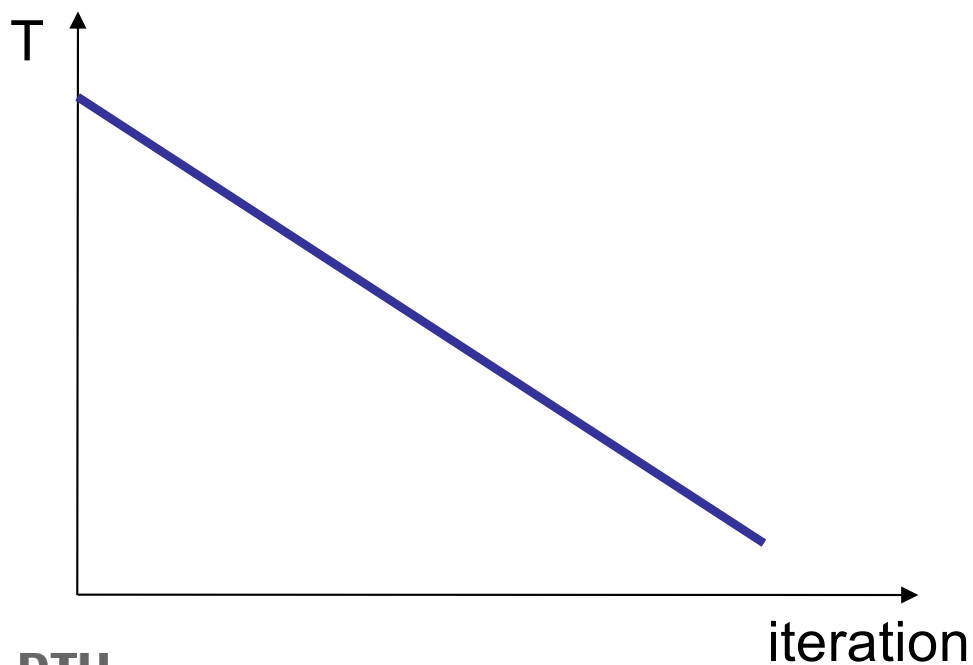
$$P = \min\left[1, \exp\left(\frac{-0.09}{0.2}\right)\right] = \min[1, 0.638] = 0.638$$

**Accept move with
 Prob = 63.8%**

Gibbs sampling - sequence alignment

What is the MC temperature?

it's a scalar decreased during the simulation

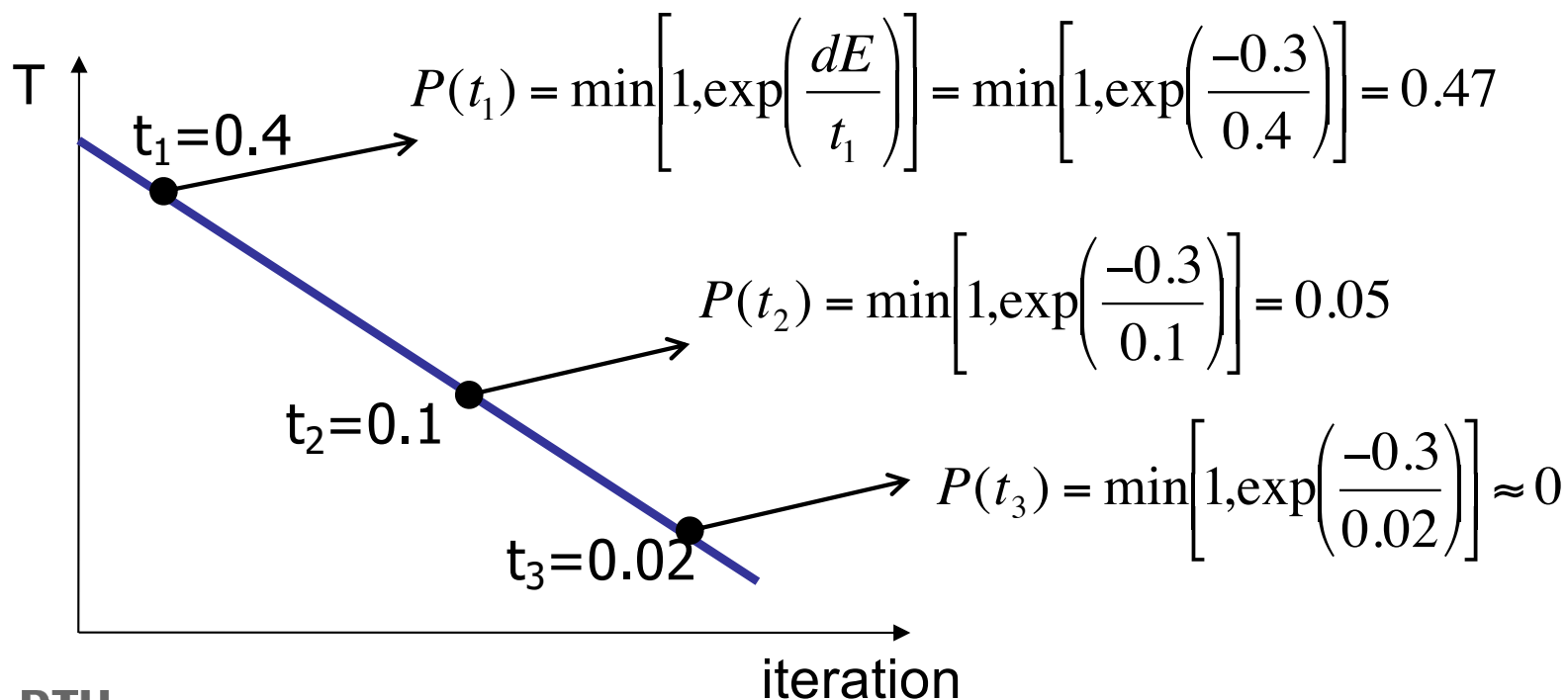


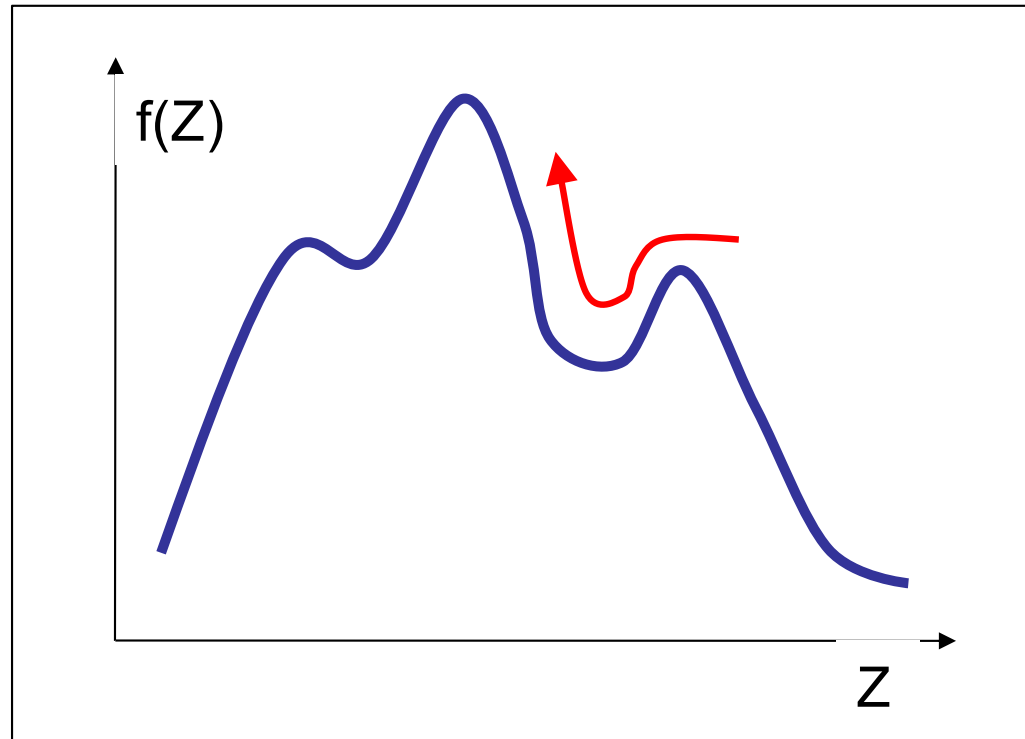
Gibbs sampling - sequence alignment

What is the MC temperature?

it's a scalar decreased during the simulation

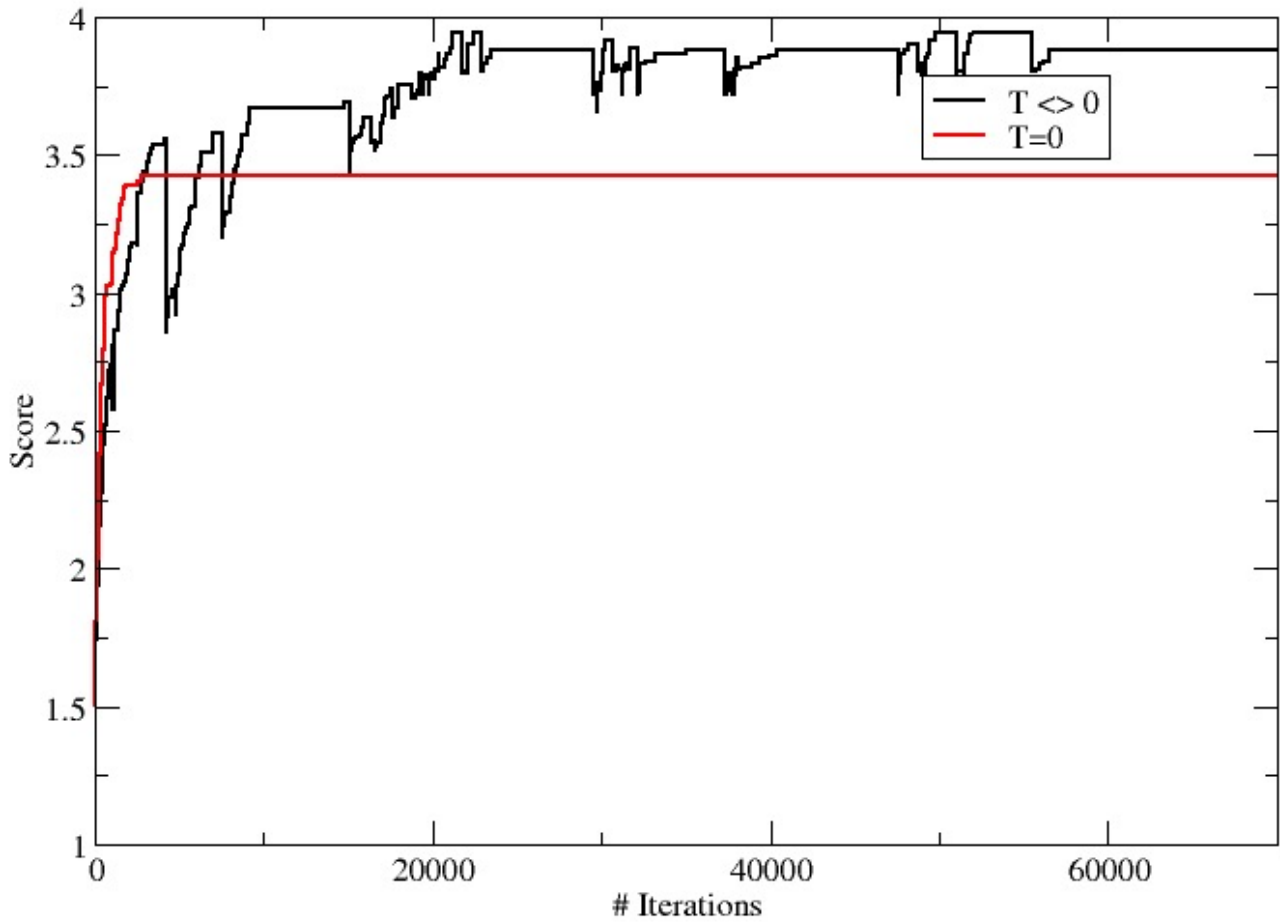
E.g. same $dE = -0.3$ but at different temperatures





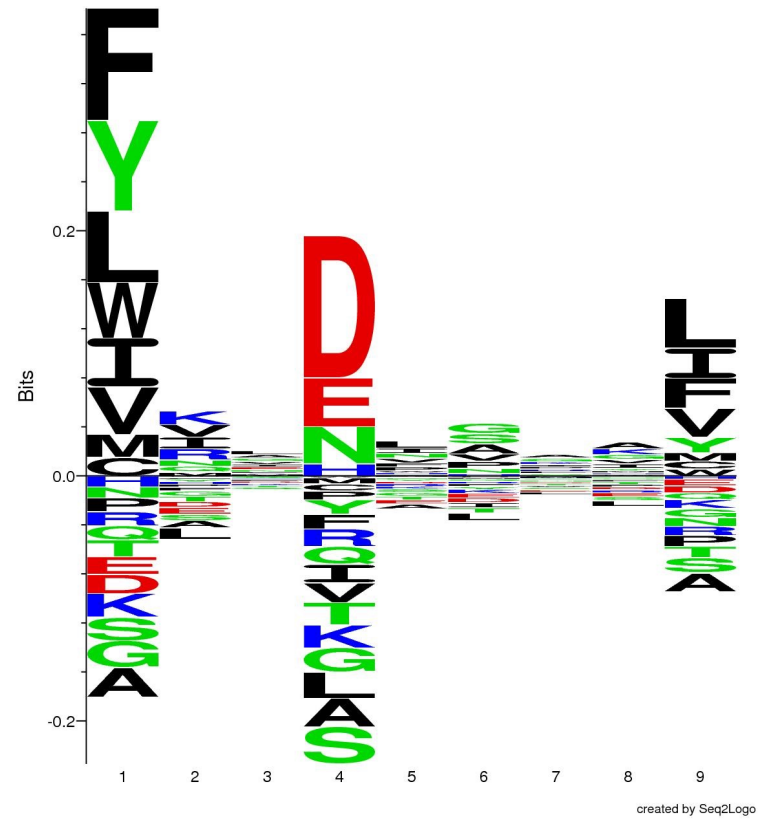
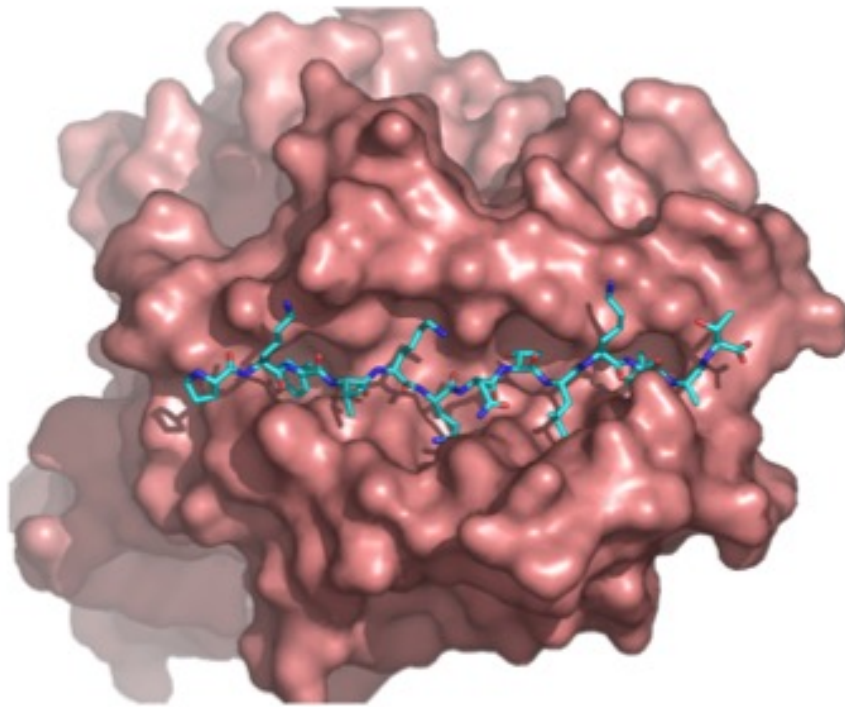
Move freely around states when the system is "warm", then cool it off to force it into a state of high fitness

Local minima

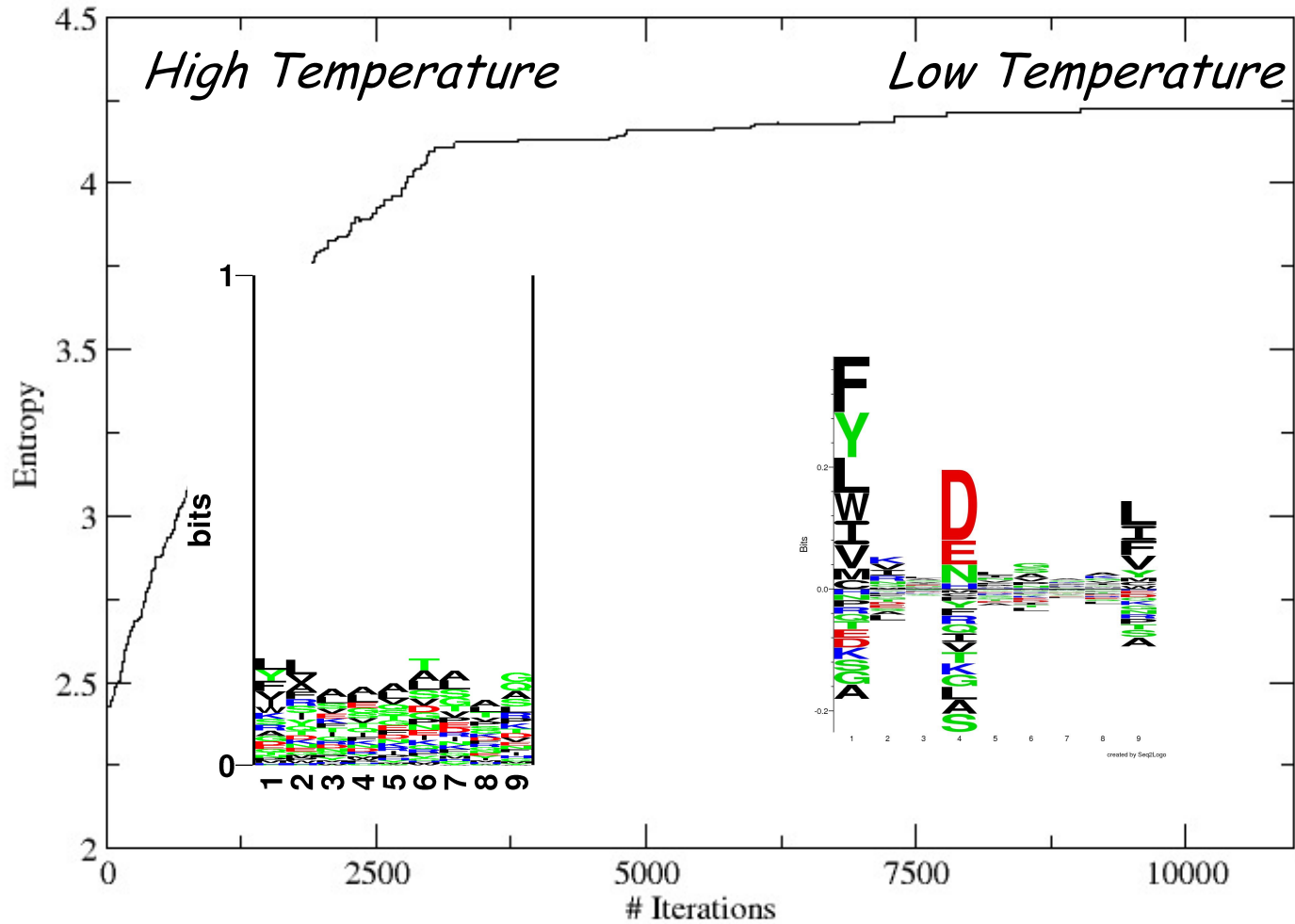


Does it work?

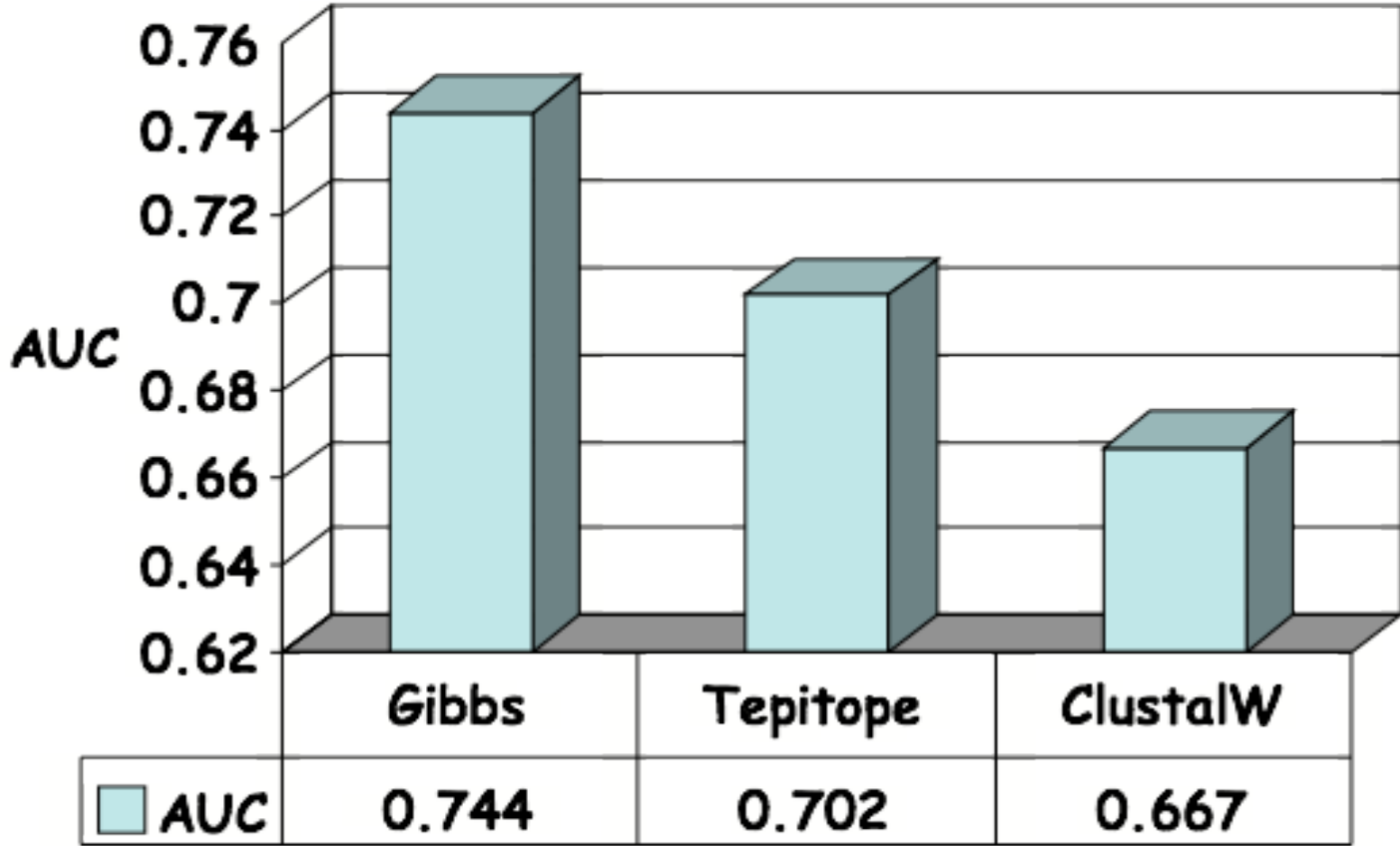
HLA-DRB3*01:01



It works

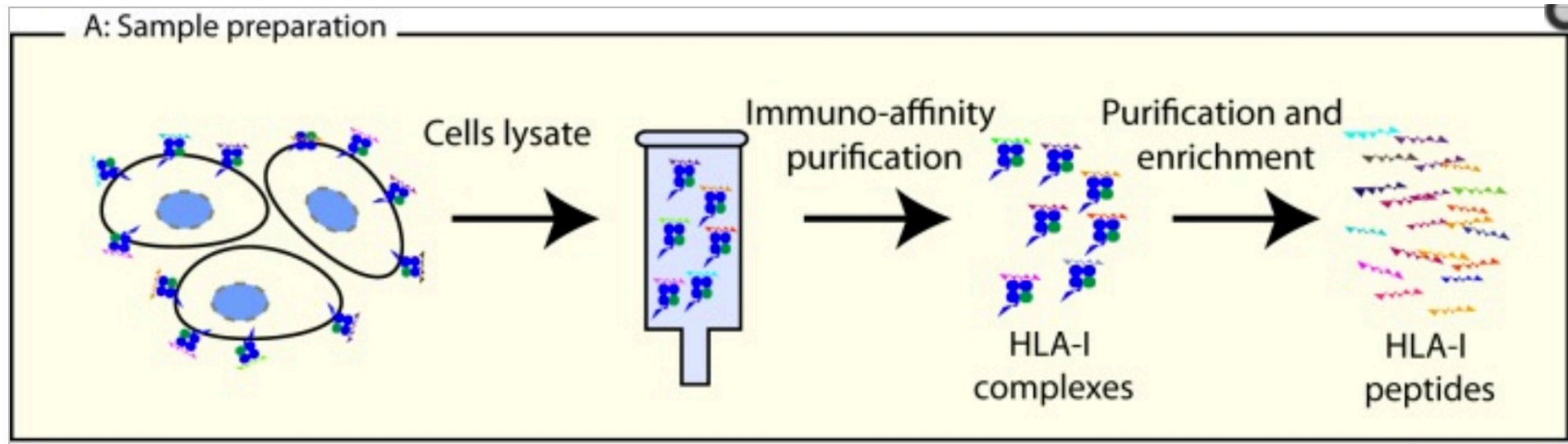


Gibbs sampler. Prediction accuracy



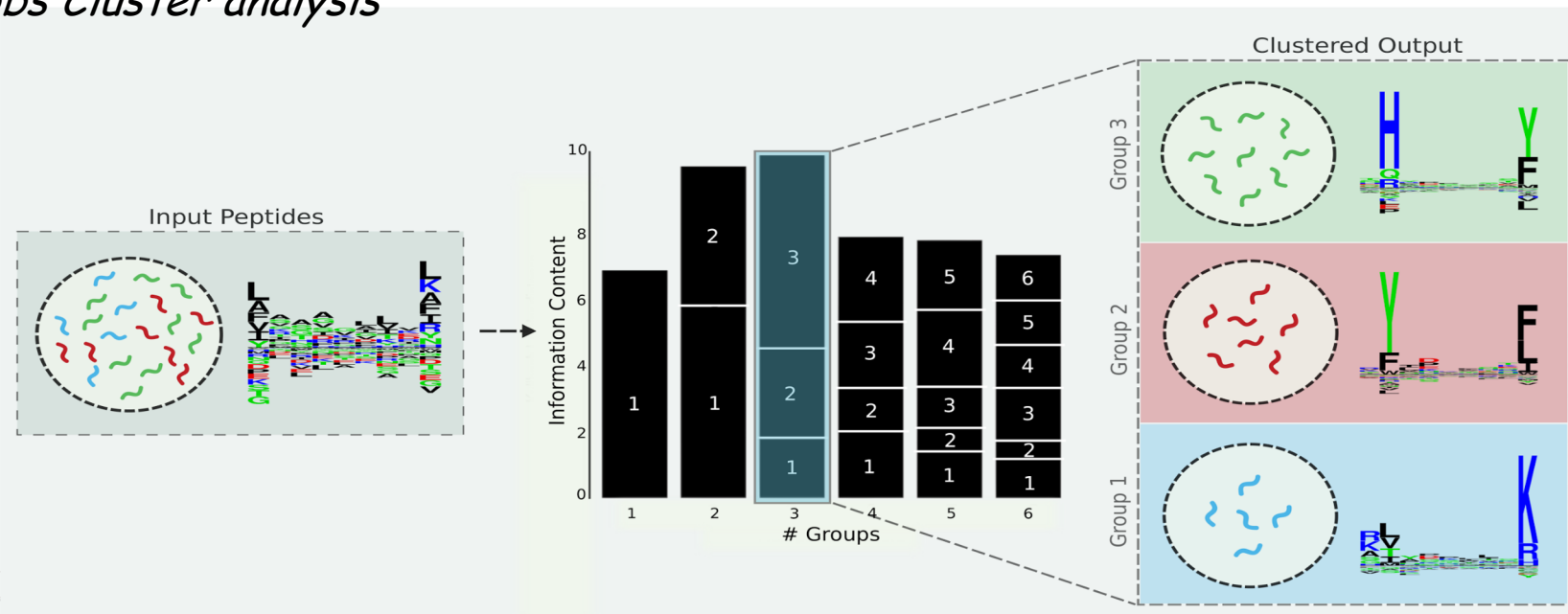
CENTER FOR BIOLOGICAL SEQUENCE ANALYSIS

Interpreting and benefitting from MS eluted ligand data sets



Gibbs Cluster analysis

Michel Deggeri, Stenbom et al, MCP, 2015



The algorithm

1. List of peptides

SLFIGLKGDIRESTV
DGEEEVQLIAAVPGK
VFRLKGGAPIKGVTF
SFSCIAIGIITLYLG
IDQVTIAGAKLRSLN
WIQKETLVTFKNPHAKKQDV
KMLLDNINTPEGIIP
ELLEFHYYLSSKLNK
LNKFISPKSVAGRFA
ESLHNPYPDYHWLRT
NKVKSLRILNTRRKL
MMGMFNMLSTVLGVS
AKSSPAYPSVLGQTI
RHLI FCHSKKCCDELA AK

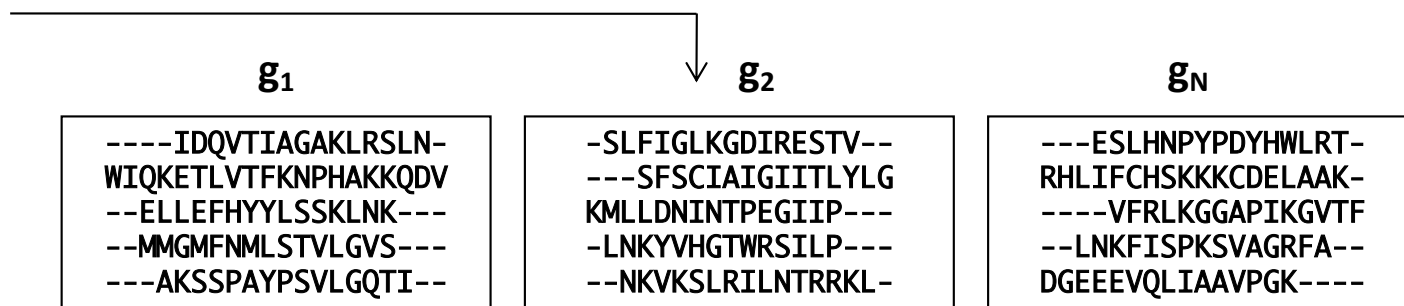
The algorithm

1. List of peptides

```

SLFIGLKGDIRESTV
DGEEEVQLIAAVPGK
VFRLKGGAPIKGVTF
SFSCIAIGIITLYLG
IDQVTIAGAKLRSLN
WIKETLVTFKNPHAKKQDV
KMLLDNINTPEGIIP
ELLEFHYYLSSKLNK
LNKFISPKSVAGRFA
ESLHNPYPDYHWLRT
NKVKSRLILNTRRKL
MMGMFNMLSTVLGVS
AKSSPAYPSVLGQTI
RHLIFCHSKKKCELAAK
    
```

2. create N random groups



3

Simple shift

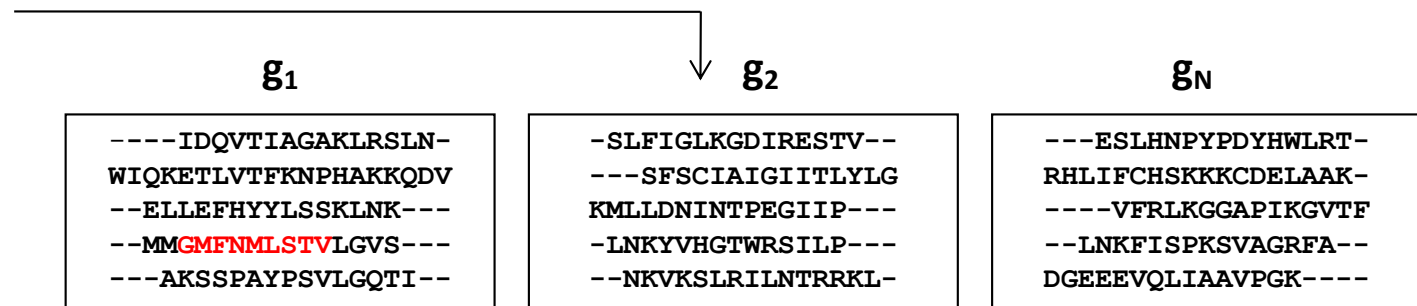
Remove peptide

The algorithm

1. List of peptides

```
SLFIGLKGDIRESTV
DGEDEVQLIAAVPGK
VFRLKGGAPIKGVTF
SFSCIAIGIITLYLG
IDQVTIAGAKLRSLN
WIQKETLVTFKNPHAKKQDV
KMLLDNINTPEGIIP
ELLEFHYYLSSKLNK
LNKFISPKSVAGRFA
ESLHNPYPDYHWLRT
NKVKSRLRILNTRRKL
MMGMFNMLSTVLGVS
AKSSPAYPSVLGQTI
RHLLIFCHSKKCCDELA
```

2. create N random groups



3. Finding the optimal configuration (the MC moves)

MMGMFNMLSTVLGVS

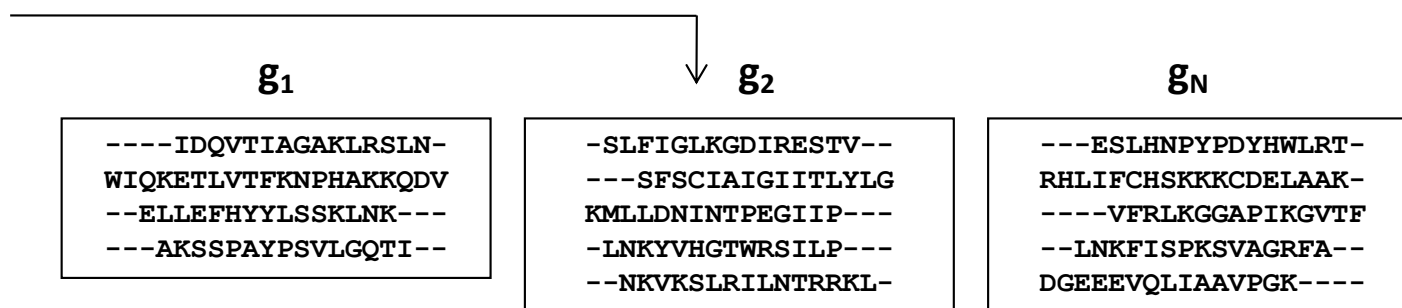
The algorithm

1. List of peptides

```

SLFIGLKGDIRESTV
DGEEEVQLIAAVPGK
VFRLKGGAPIKGVTF
SFSCIAIGIITLYLG
IDQVTIAGAKLRSLN
WIQKETLVTFKNPHAKKQDV
KMLLDNINTPEGIIP
ELLEFHYYLSSKLNK
LNKFISPKSVAGRFA
ESLHNPYPDYHWLRT
NKVKSRLRILNTRRKL
MMGMFNMLSTVLGVS
AKSSPAYPSVLGQTI
RHLLIFCHSKKCCDELAAK
    
```

2. create N random groups



3. Finding the optimal configuration (the MC moves)

MMGMFNMLSTVLGVS

5. Score new core to log-odds matrices

$$dE = E_{new} - E_{old}$$

4. Random shift of the core

MMGMFNMLSTVLGVS

6. Accept or reject move

$$P = \min \left[1, \exp \left(\frac{dE}{T} \right) \right]$$

The algorithm

1. List of peptides

```
SLFIGLKGDIRESTV
DGEDEVQLIAAVPGK
VFRLKGGAPIKGVTF
SFSCIAIGIITLYLG
IDQVTIAGAKLRSLN
WIKETLVTFKNPHAKKQDV
KMLLDNINTPEGIIP
ELLEFHYYLSSKLNK
LNKFISPKSVAGRFA
ESLHNPYPDYHWLRT
NKVKSRLILNTRRKL
MMGMFNMLSTVLGVS
AKSSPAYPSVLGQTI
RHLIFCHSKKCCDELAAK
```

2. create N random groups

g_1

```
----IDQVTIAGAKLRSLN-
WIKETLVTFKNPHAKKQDV
--ELLEFHYYLSSKLNK---
--MMGMFNMLSTVLGVS---
---AKSSPAYPSVLGQTI--
```

g_2

```
-SLFIGLKGDIRESTV--
---SFSCIAIGIITLYLG
KMLLDNINTPEGIIP---
-LNKYVHGTWRSILP---
--NKVKSRLILNTRRKL-
```

g_N

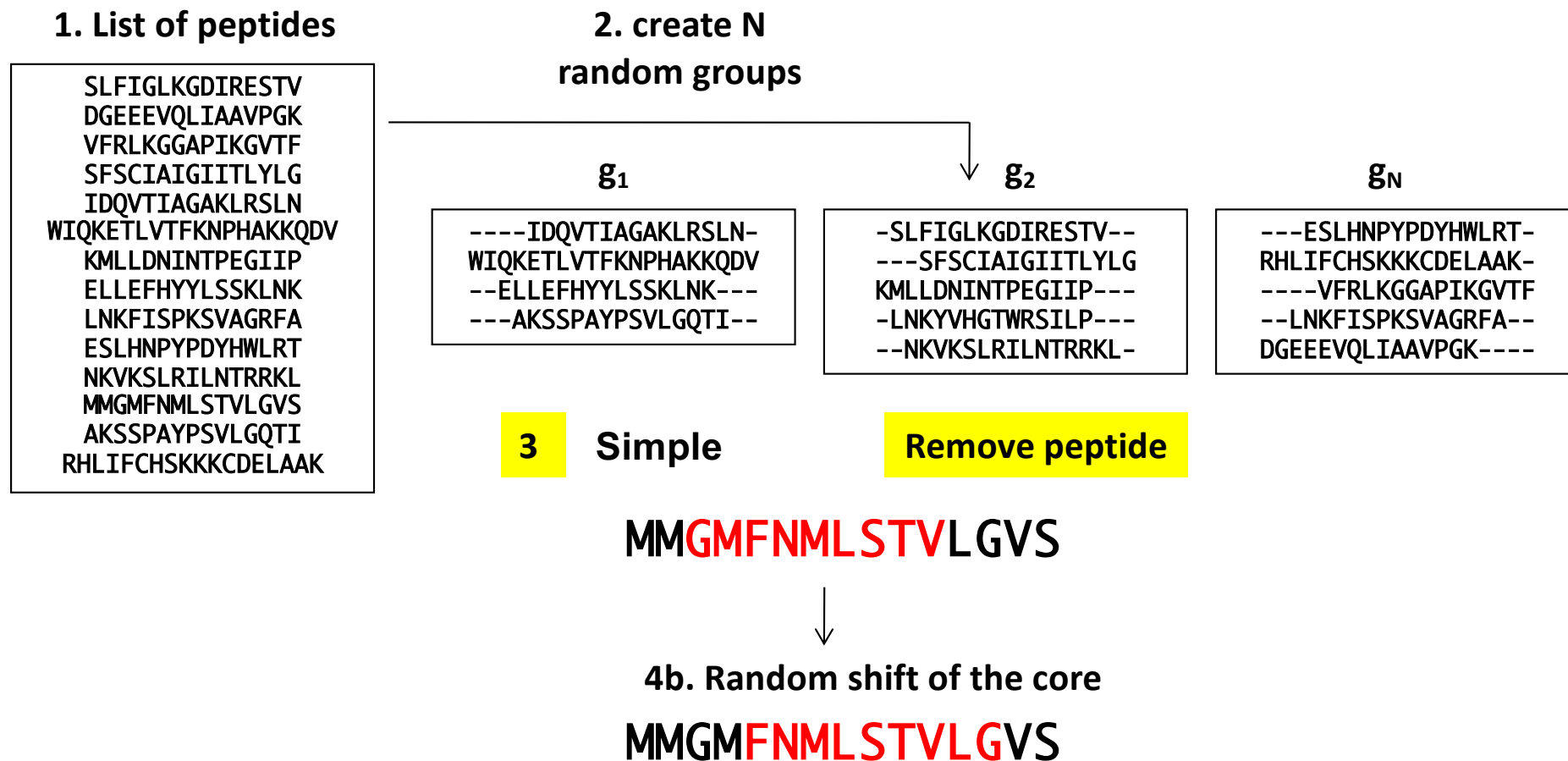
```
---ESLHNPYPDYHWLRT-
RHLIFCHSKKCCDELAAK-
----VFRLKGGAPIKGVTF
--LNKFISPKSVAGRFA--
DGEDEVQLIAAVPGK----
```

3 Simple

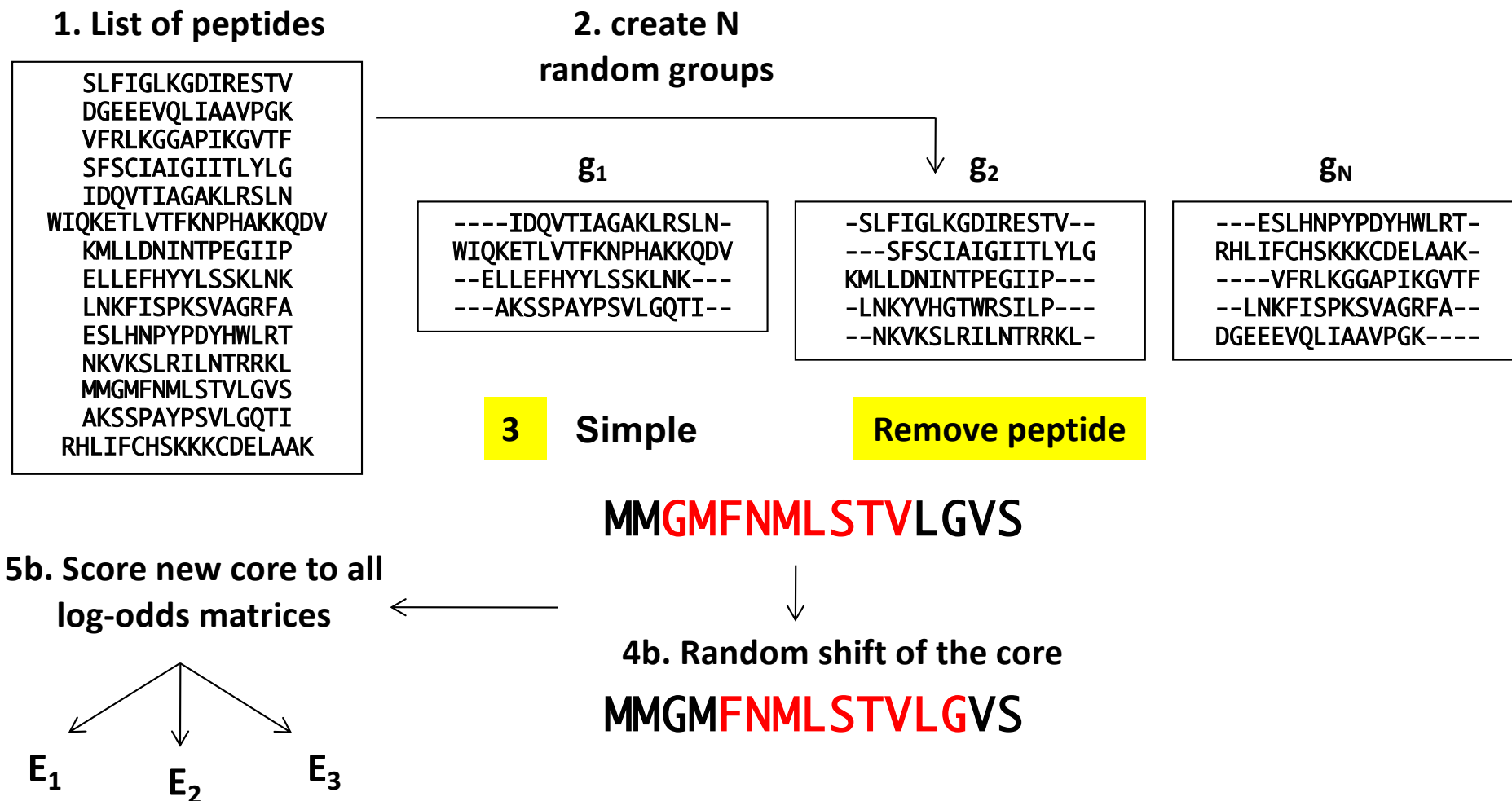
Remove peptide

MMGMFNMLSTVLGVS

The algorithm

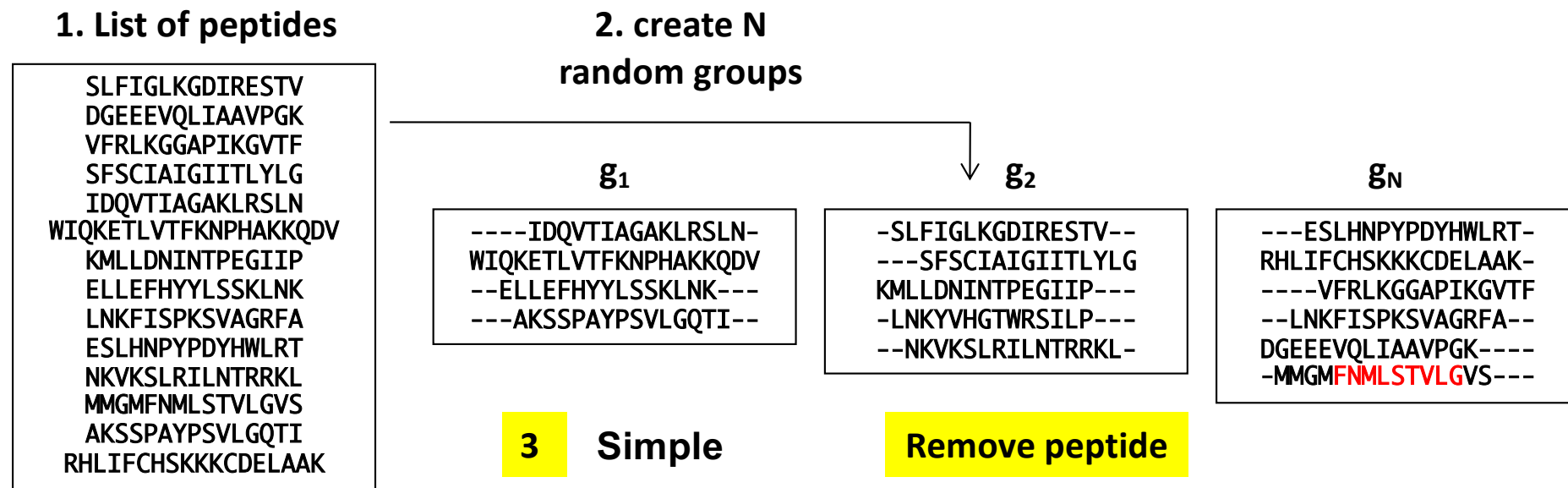


The algorithm

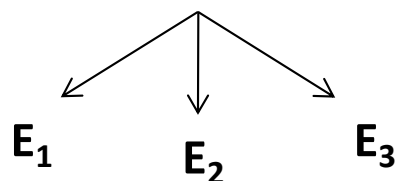


$$dE = E_{before} - \max(E_1, E_2, E_3)$$

The algorithm



5b. Score new core to all log-odds matrices



MMGMFNMLSTVLGVS

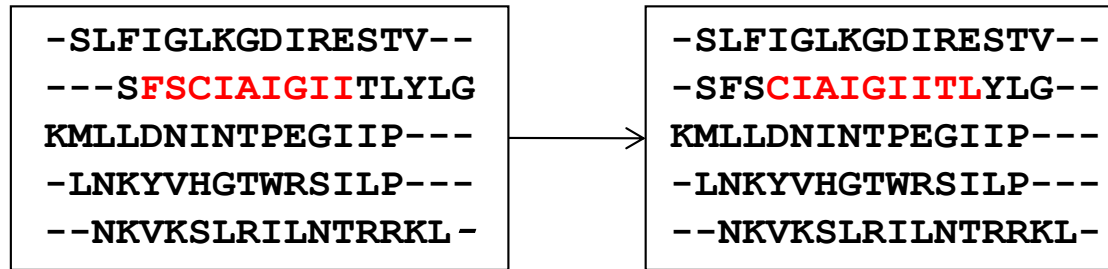
4b. Random shift of the core

MMGMFNMLSTVLGVS

6b. Accept or reject move

$$dE = E_{before} - \max(E_1, E_2, E_3) \quad P = \min\left[1, \exp\left(\frac{dE}{T}\right)\right]$$

The scoring function



$$LO_{A,j} = \frac{n}{n + \sigma} \log \frac{P_{A,j}}{q_A}$$

Avoid small specialized clusters ($\sigma = 10$)

$$E = \sum_j LO_{A,j}$$

$$E_i^* = E_i - \lambda \max_{\substack{1 \leq n \leq g \\ n \neq i}} (E_n, 0)$$

Maximize intra cluster similarity whilst minimize inter cluster similarity

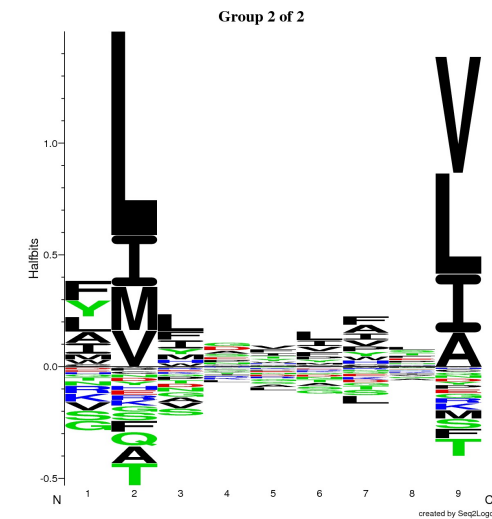
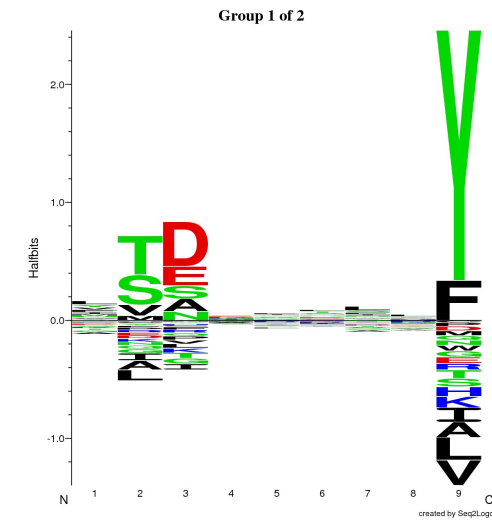
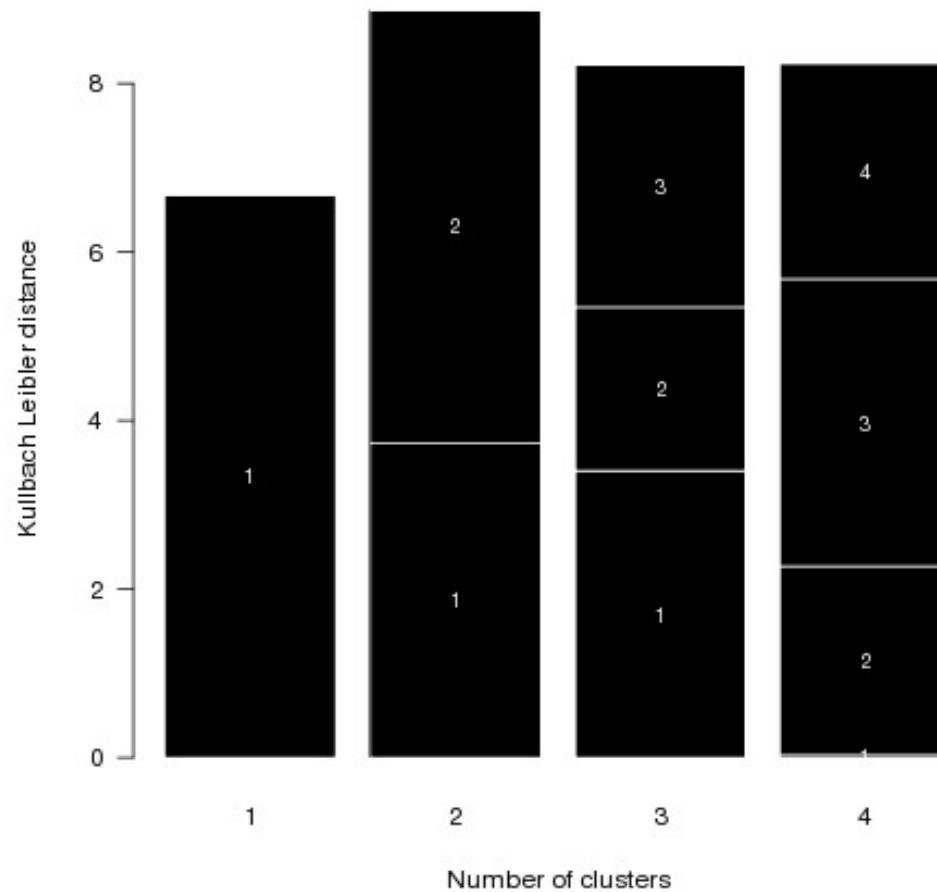
$$dE = E_{before}^* - E_{after}^*$$

$$P = \min \left[1, \exp \left(\frac{dE}{T} \right) \right]$$

P = Probability of accepting the move

A mixture of 500 9mer peptides. How many motifs?

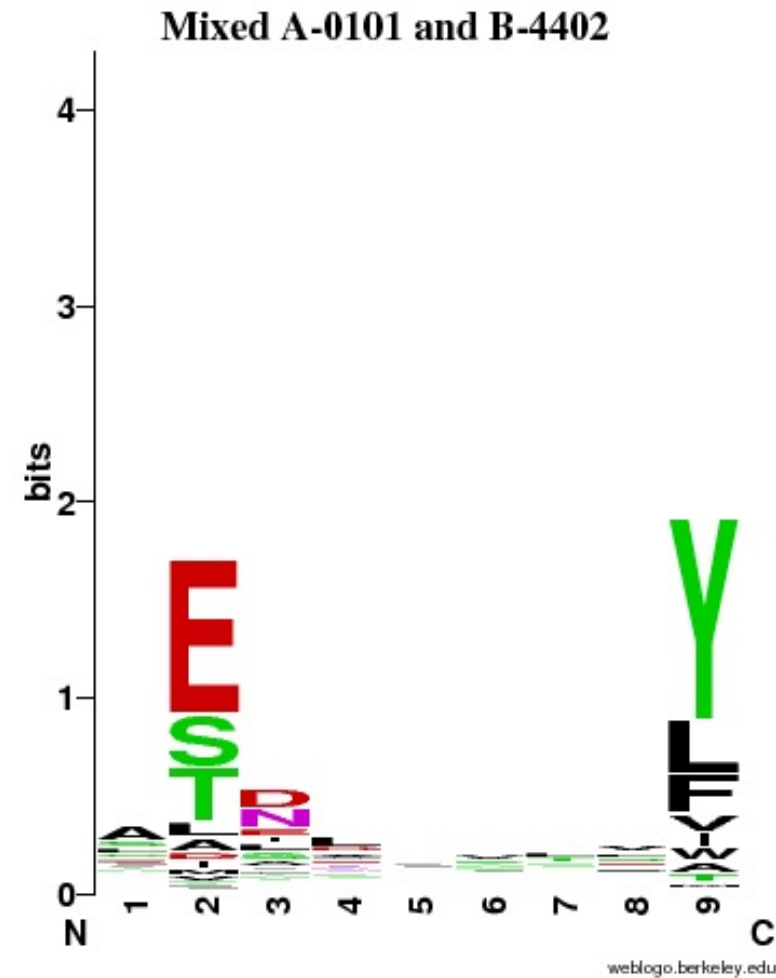
A mixture of 500 9mer peptides. How many motifs?



Two MHC class I alleles: HLA-A*0101 and HLA-B*4402

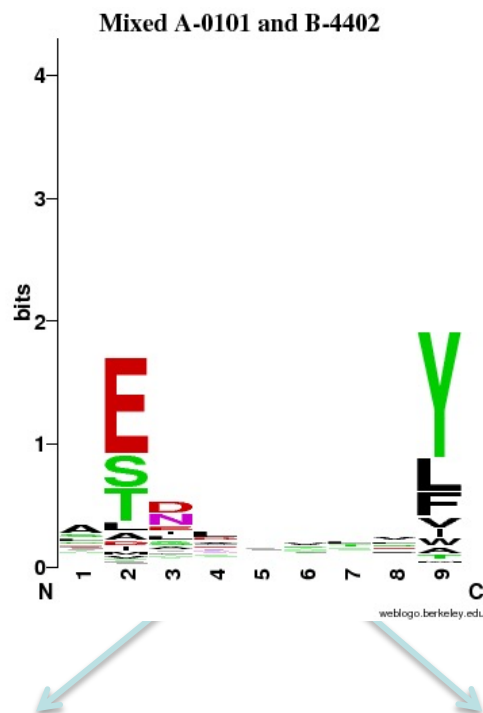
Mixture of 100 binders
for the two alleles

ATDKAAAAY	A*0101
EVDQTKIQY	A*0101
AETGSQGVY	B*4402
ITDITKYLY	A*0101
AEMKTDAAT	B*4402
FEIKSAKKF	B*4402
LSEMLNKEY	A*0101
GELDRWEKI	B*4402
LTDSSTLLV	A*0101
FTIDFKLKY	A*0101
TTTIKPVSY	A*0101
EEKAFSPEV	B*4402
AENLWVPVY	B*4402



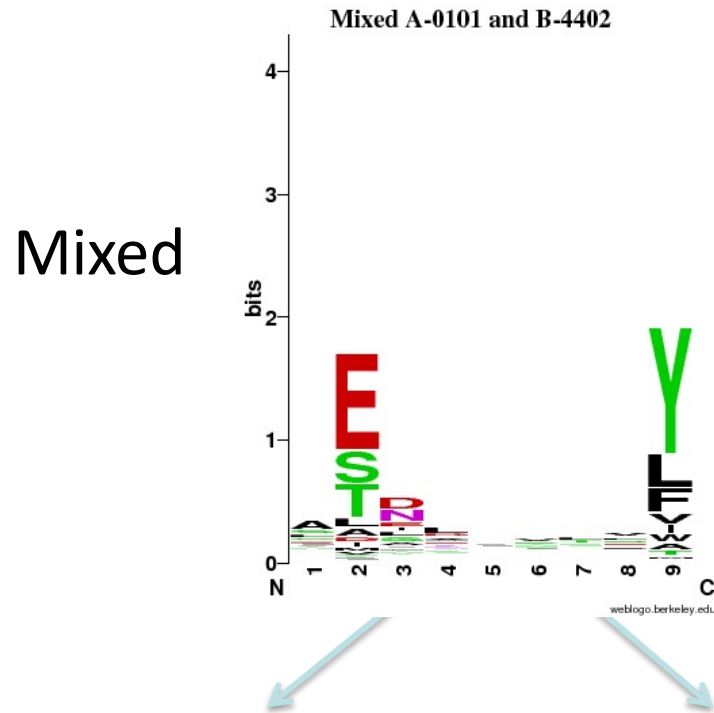
Two MHC class I alleles: HLA-A*0101 and HLA-B*4402

Mixed

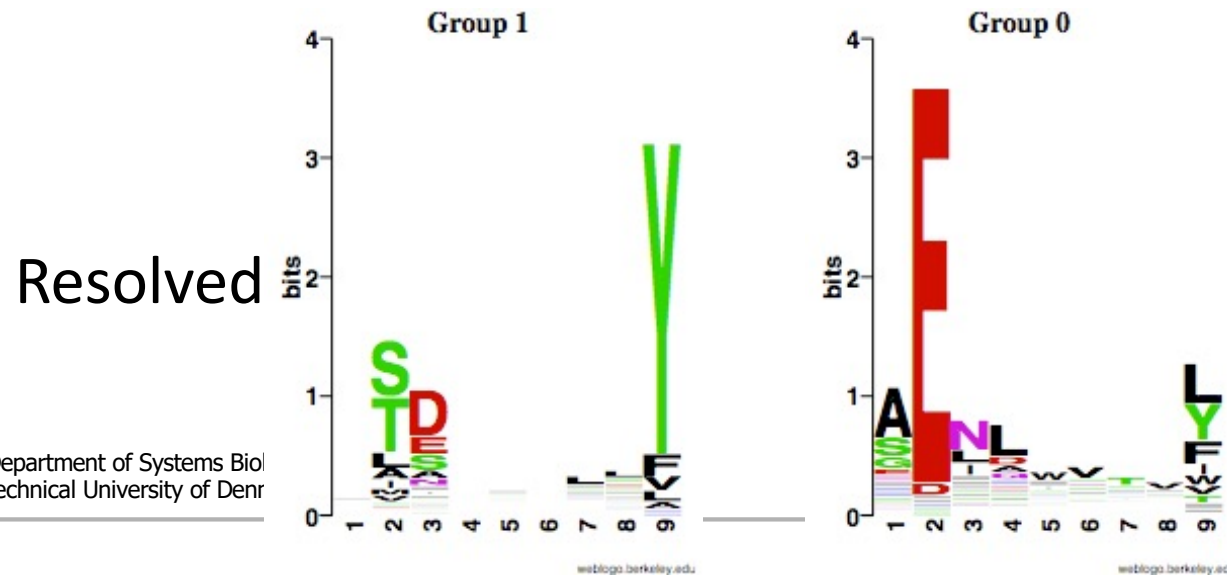


	A0101	B4402
G 1		
G 2		

Two MHC class I alleles: HLA-A*0101 and HLA-B*4402

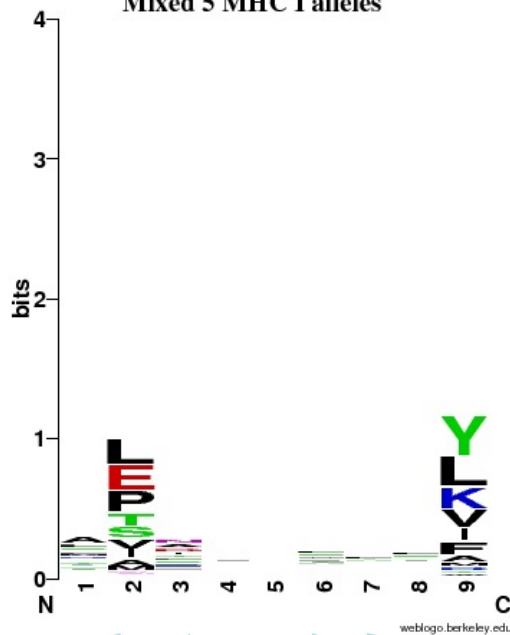


	A0101	B4402
G 1	97	3
G 2	3	97



Five MHC class I alleles

Mixed 5 MHC I alleles



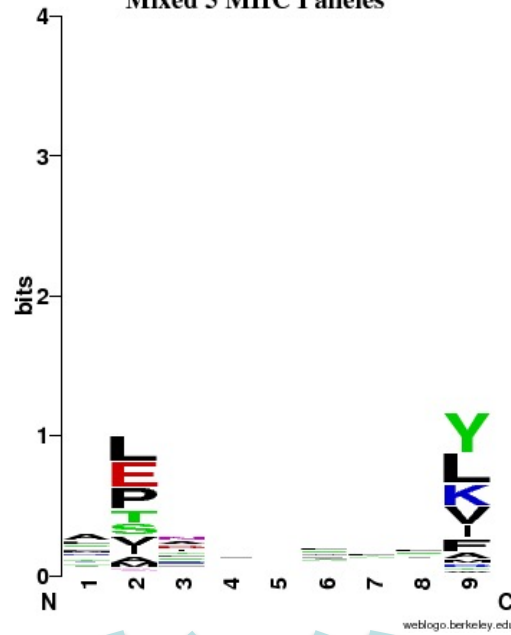
	A010	A020	A030	B0702	B4402
G 0		1	1		
G 1					
G 2					
G 3					
G 4					

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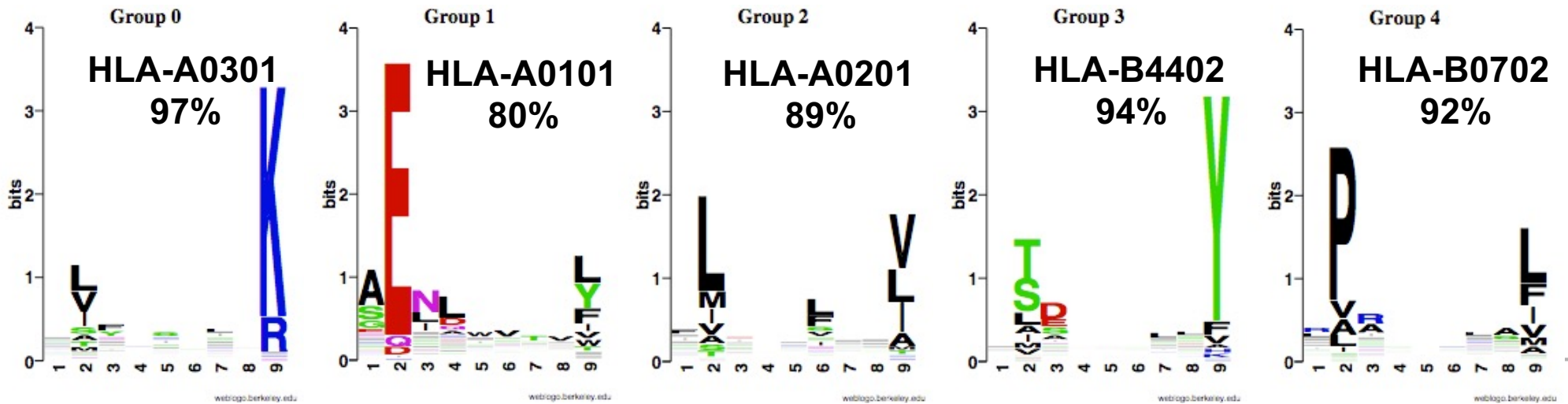


Five MHC class I alleles

Mixed 5 MHC I alleles



	A010	A020	A030	B0702	B4402
G 0	0	1	76	1	0
G 1	2	4	0	0	95
G 2	5	87	5	1	0
G 3	93	2	19	0	2
G 4	0	6	0	98	3

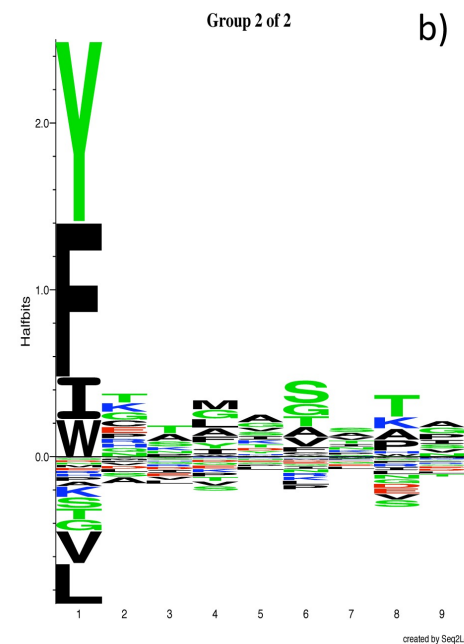
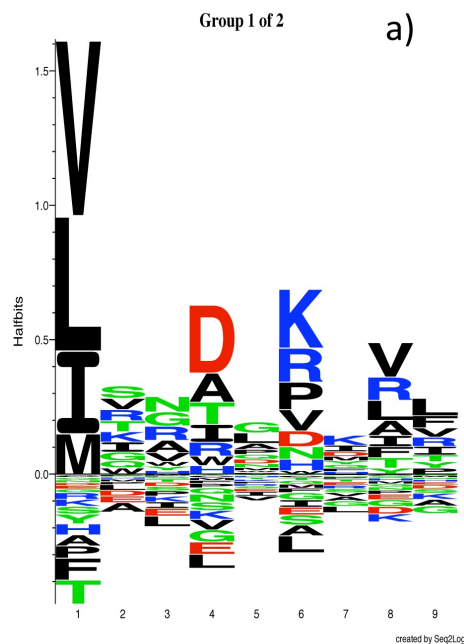


Adding in alignment (MHC class II)

SLFIGLKGDIRESTV
 DGEEEVQLIAAVPGK
 VFRLKGGAPIKGVTF
 SFSCIAIGIITLYLG
 IDQVTIAGAKLRSLN
 WIQKETLVTFKNPHAKKQDV
 KMLLDNINTPEGIIP
 ELLEFHYLSSKLNK
 LNKFISPKSVAGRFA
 ESLHNPYPDYHWLRT
 NKVKSLRILNTRRKL
 MMGMFNMLSTVLGVS
 AKSSPAYPSVLGQTI
 RHLIFCHSKKKCDELAAK

HLA-DRB1*03:01

HLA-DRB1*04:01



c)

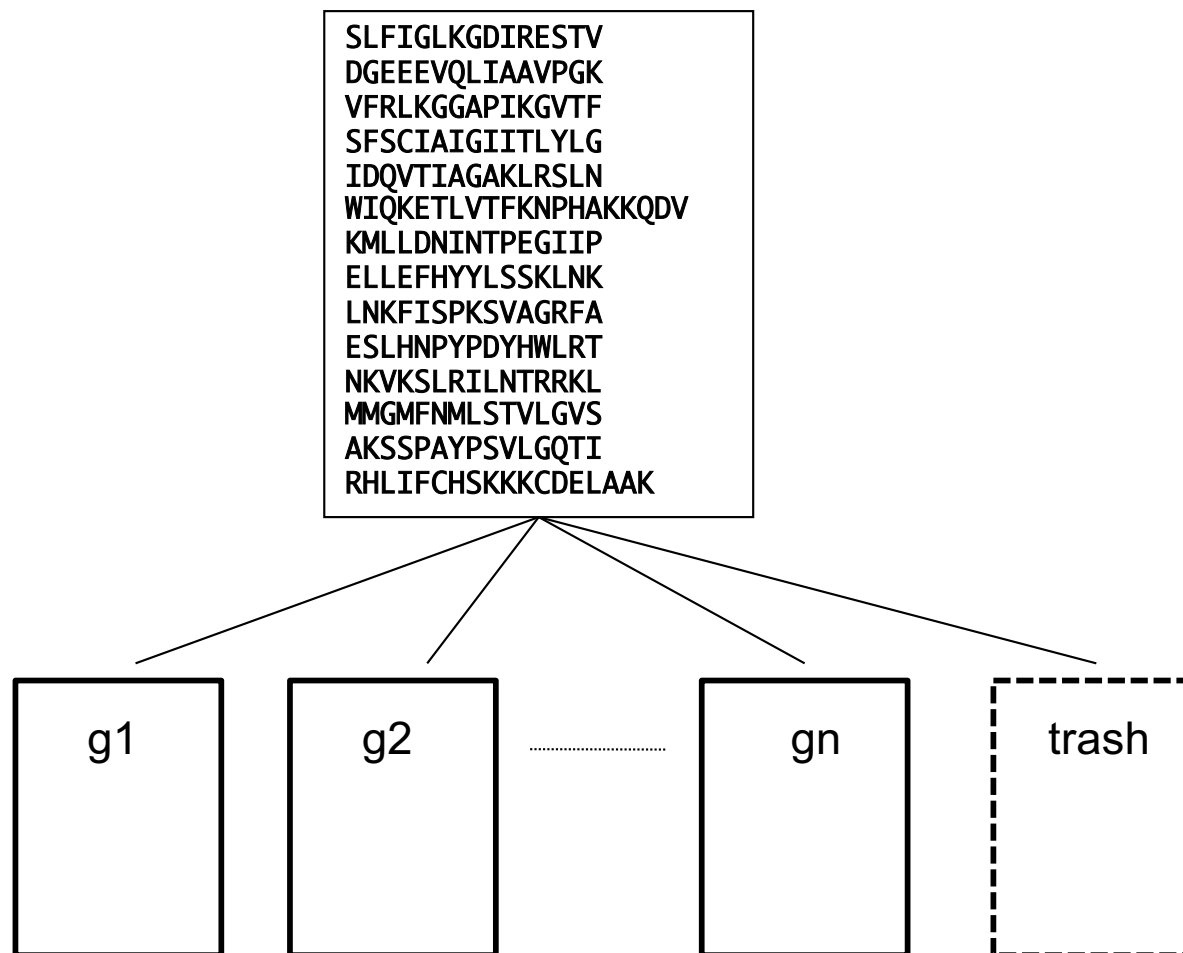
	DRB1*03:01	DRB1*04:01
C1	170	53
C2	31	149

Dealing with noisy data

- Experimental data often contain **false positives**
- Outliers do not match any recurrent motif
- Introduce a **garbage bin** to collect outliers

Dealing with noisy data

- Introduce a **garbage bin** to collect outliers



Move to the **trash cluster** peptides that do not match any motif

Dealing with noisy data

200 binders to 3 MHC
 class I alleles

50 random sequences
 are added to the data set

3 alleles	HLA-A0101	HLA-B0702	HLA-B4001	Random	
g0	0	197	2	6	205
g1	199	1	0	2	202
g2	0	0	196	5	201
trash	1	2	2	37	42
	200	200	200	50	

Dealing with noisy data

200 binders to 3 MHC class I alleles

50 random sequences are added to the data set

3 alleles	HLA-A0101	HLA-B0702	HLA-B4001	Random	
g0	0	197	2	6	205
g1	199	1	0	2	202
g2	0	0	196	5	201
trash	1	2	2	37	42
	200	200	200	50	

DHHFTPQII

NAFGWENAY
 SQTSYQYLI

ELPIVTPAL
 ADKNLIKCS

Dealing with noisy data

Table 1: Measured, predicted and re-tested binding affinities (in nM) for peptides assigned to the trash cluster.

Peptide	HLA	IEDB ^a	Predicted ^b	Validated ^c
DHHFTPQII	A*01:01	62	28485	24822
SQTSYQYLI	B*07:02	248	24349	49928
NAFGWENAY	B*07:02	350	24481	-
TVFKGFVNK	B*27:05	235	13723	-
ELPIVTPAL	B*40:01	314	15208	-
ADKNLIKCS	B*40:01	316	33324	76190

^a Binding affinity deposited in the Immune Epitope Database.

^b Predicted binding affinities using NetMHCcons.

^c Re-tested binding affinities after detection as outliers.

As a rule of thumb, generally affinity < 50nM identifies a strong binder, 50nM < affinity < 500nM a weak binder, affinity > 500nM non-binders.

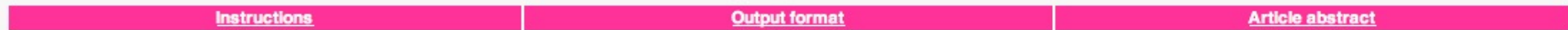


CBS >> [CBS Prediction Servers](#) >> GibbsCluster-1.0

GibbsCluster-1.0 Server

Simultaneous alignment and clustering of peptide data

View the [version history](#) of this server. All the previous versions are available online, for comparison and reference.



DATA SUBMISSION

Paste peptides in the box:

or submit a file directly from your local disk:

no file selected

Sample data: [Sample 1](#) - [Sample 2](#)

SUBMIT job