DTU

DTU Health Technology
Bioinformatics

Brief refresher on conditional probabilities and the Bayesian theorem

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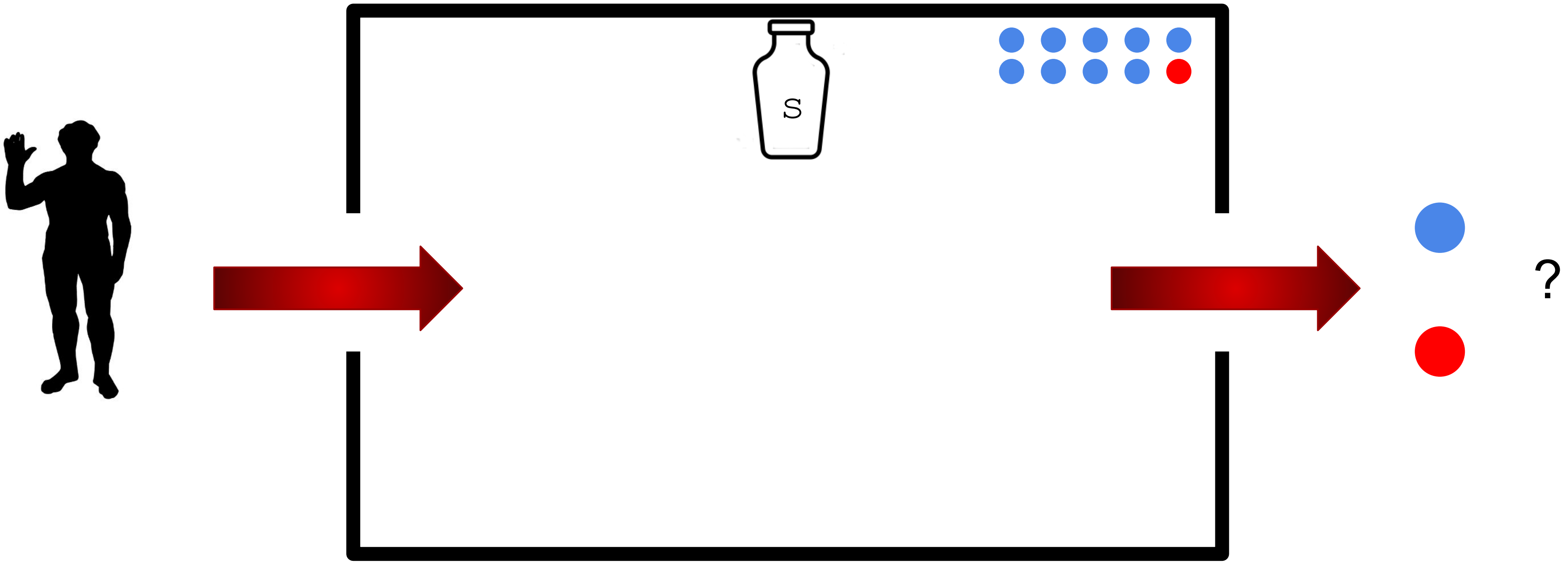
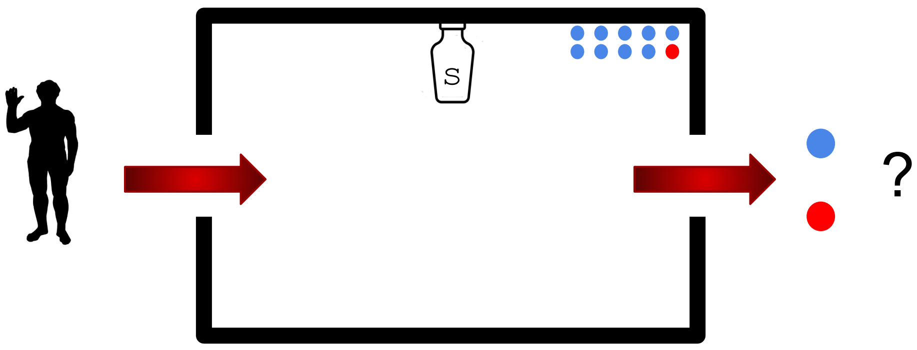
Brief probability reminder … but first a little game!



Brief probability reminder

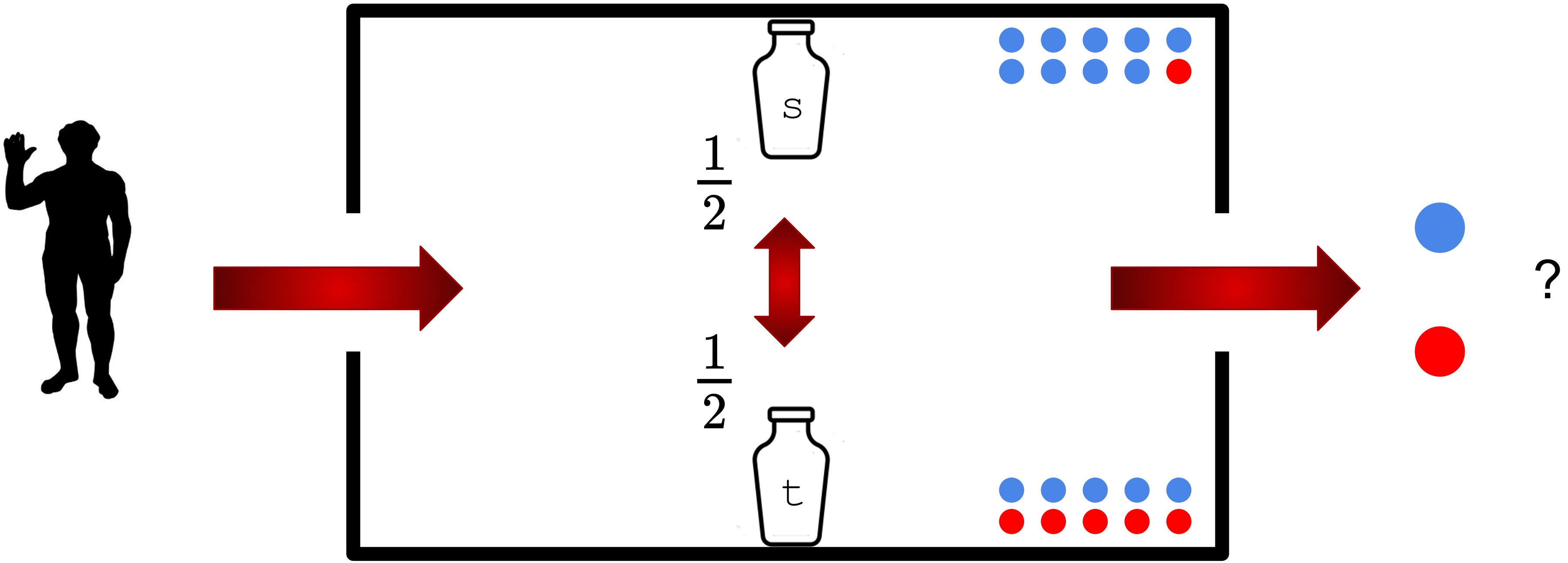


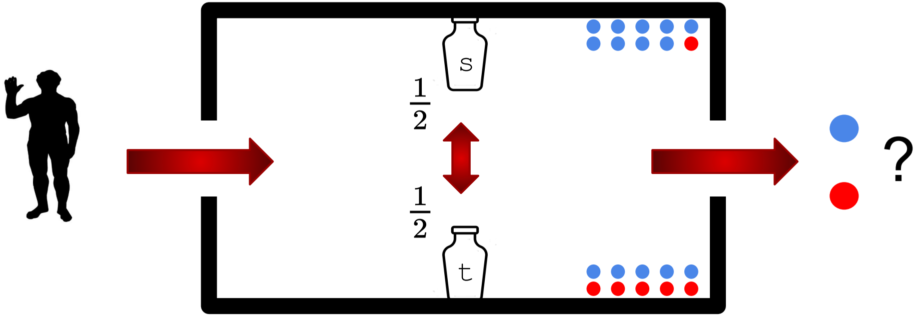
Events:

$$E = \text{our player picked a red ball}$$

$$P(E) = \frac{1}{10} = 0.1$$

Brief probability reminder





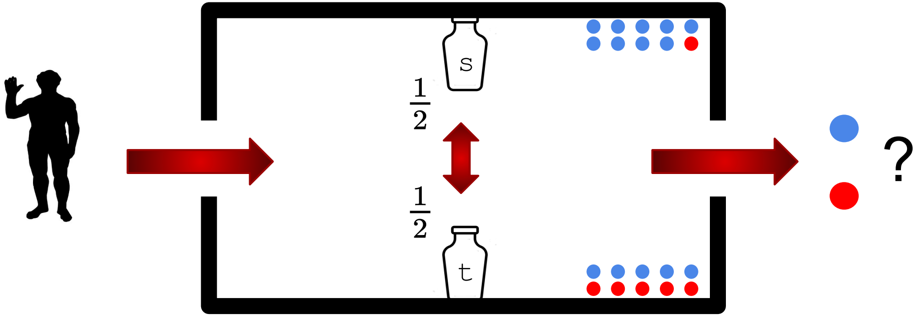
E = our player picked a red ball

S = our player picked the ‘s’ urn

$$P(S) = \frac{1}{2}$$

$$P(E|S) = \frac{1}{10} = 0.1$$

conditional probability (assuming our player picked the ‘s’ urn)



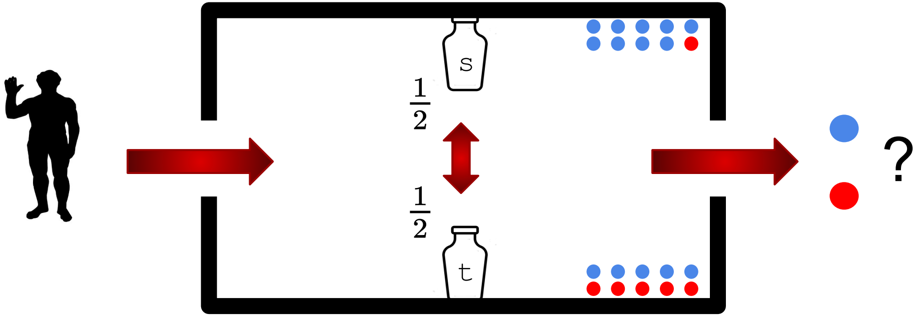
E = our player picked a red ball

S = our player picked the 's' urn

T = our player picked the 't' urn

$$P(T) = \frac{1}{2}$$

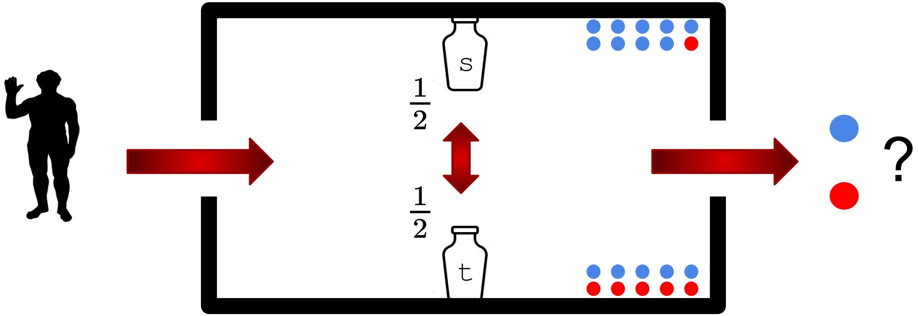
$$P(E|T) = \frac{5}{10} = \frac{1}{2} = 0.5$$



S $\quad\quad\quad\quad\quad$ T

$P(E) =$ (Our player picked urn ‘s’ and picked a red ball) + (Our player picked urn ‘t’ and picked a red ball)

$$P(E) = P(S)P(E|S) + P(T)P(E|T)$$

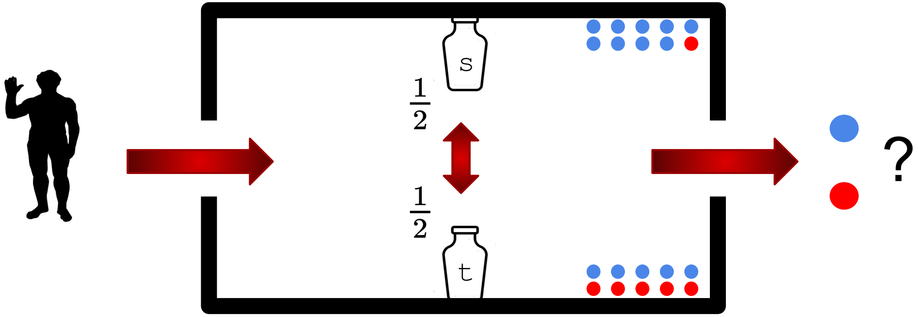


$$P(E) = P(S)P(E|S) + P(T)P(E|T)$$

$$P(E) = \frac{1}{2} \quad \frac{1}{10} + \frac{1}{2} \quad \frac{5}{10}$$

$$P(E) = \frac{1}{20} + \frac{5}{20}$$

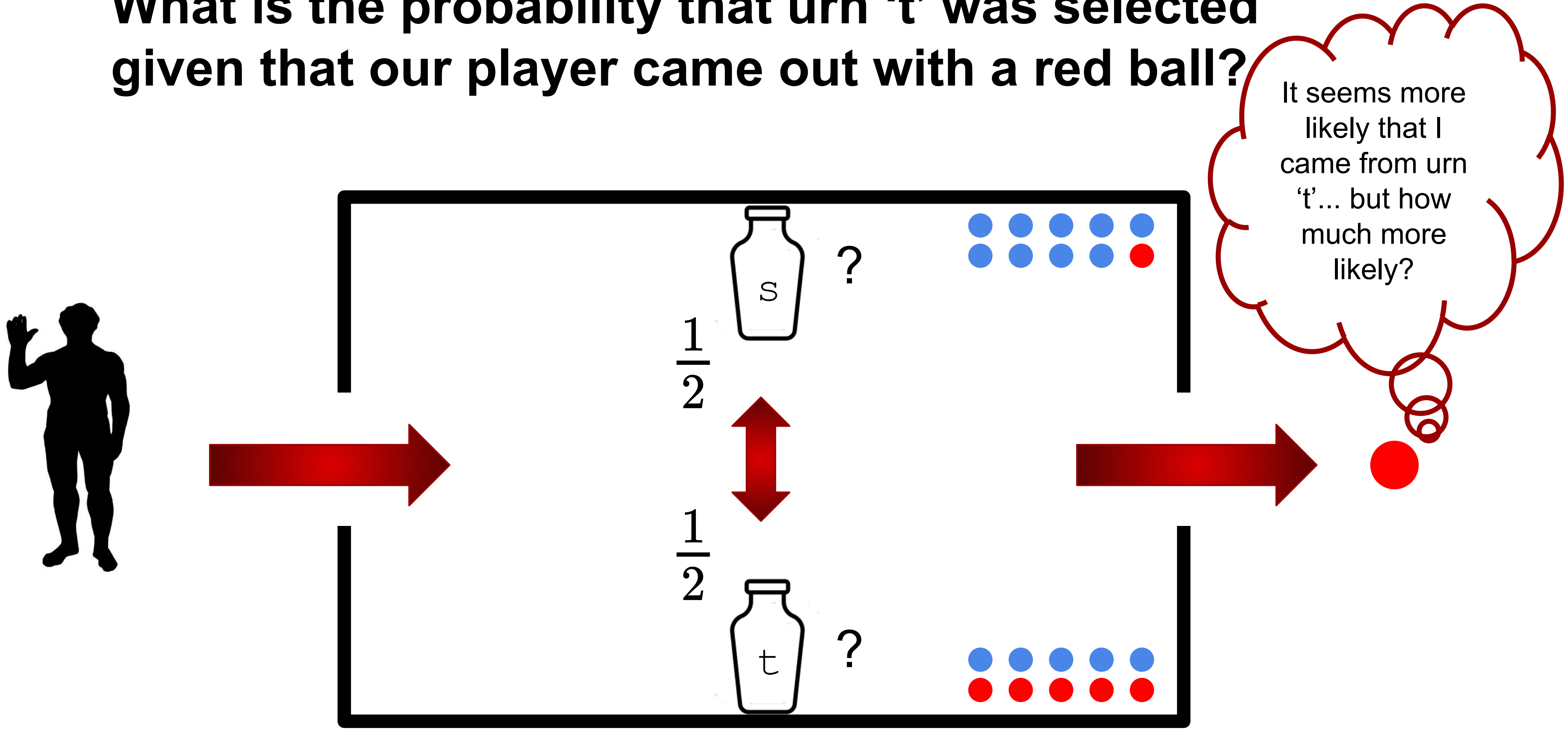
$$P(E) = \frac{6}{20}$$

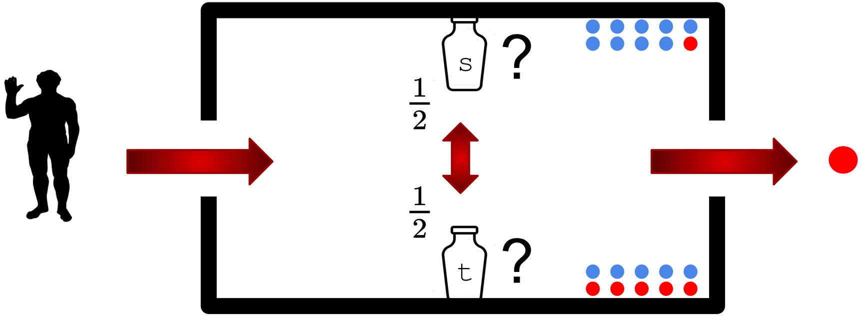


$$P(E) = \frac{6}{20}$$

There is a 30% chance of getting a red ball

What is the probability that urn 't' was selected given that our player came out with a red ball?



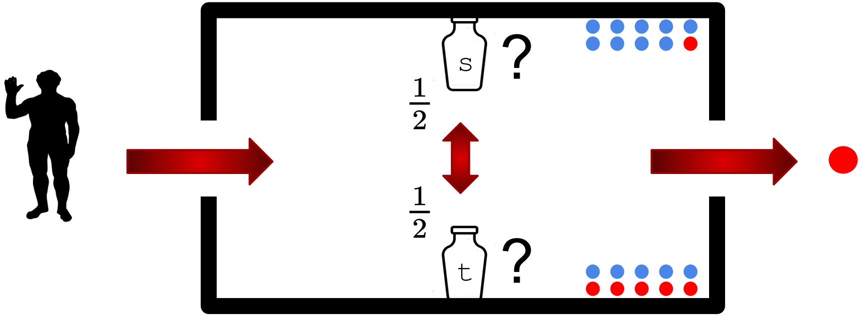


E = our player picked a red ball

T = our player picked the ‘t’ urn

We seek:

$$P(T|E)$$



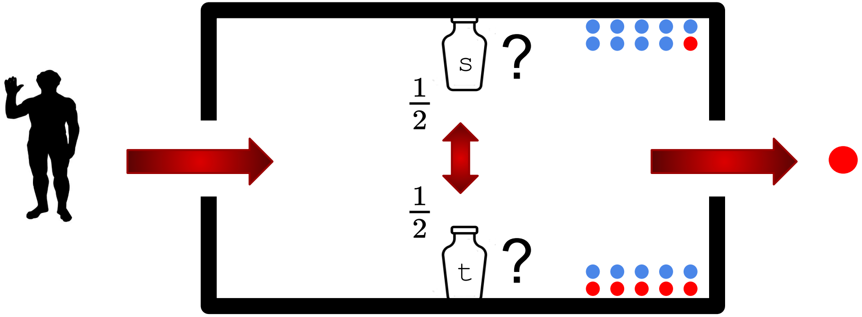
E = our player picked a red ball

T = our player picked the 't' urn

Bayes' Theorem

Thomas Bayes (1701 - 1761)

$$P(T|E) = \frac{P(T)P(E|T)}{P(E)}$$

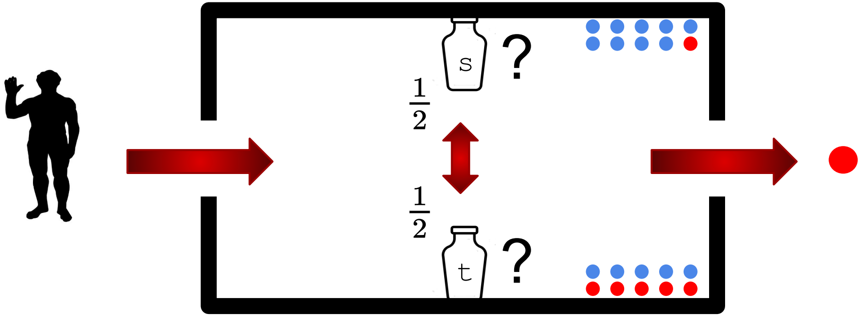


E = our player picked a red ball

T = our player picked the 't' urn

What is the **prior** probability of picking urn 't'?

$$P(T|E) = \frac{P(T)P(E|T)}{P(E)}$$

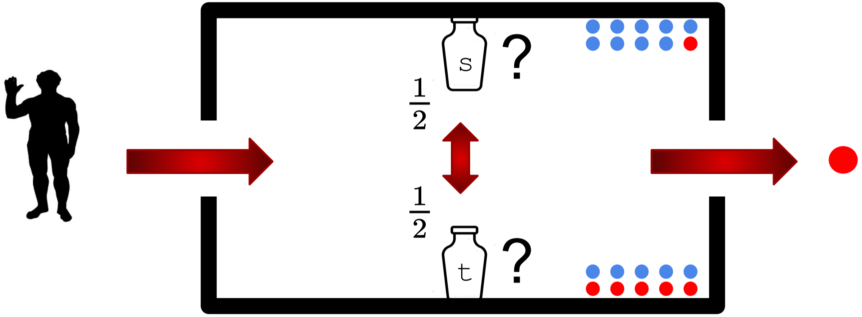


E = our player picked a red ball

T = our player picked the ‘t’ urn

What is the **prior** probability of selecting urn ‘t’?

$$P(T|E) = \frac{\frac{1}{2} \quad P(E|T)}{P(E)}$$

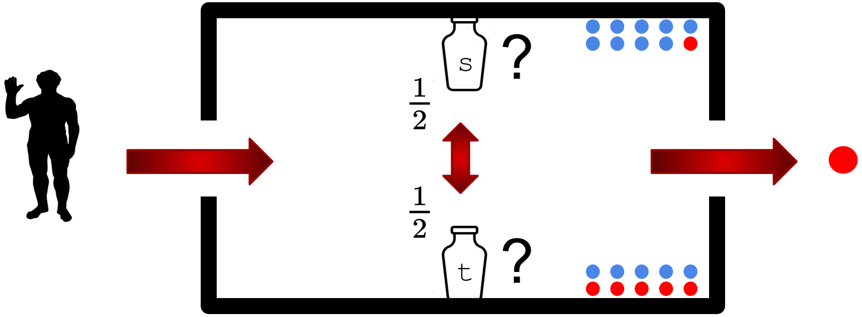


E = our player picked a red ball

T = our player picked the ‘t’ urn

What is the probability of sampling a red ball given than I selected the urn ‘t’?

$$P(T|E) = \frac{\frac{1}{2} \quad P(E|T)}{P(E)}$$

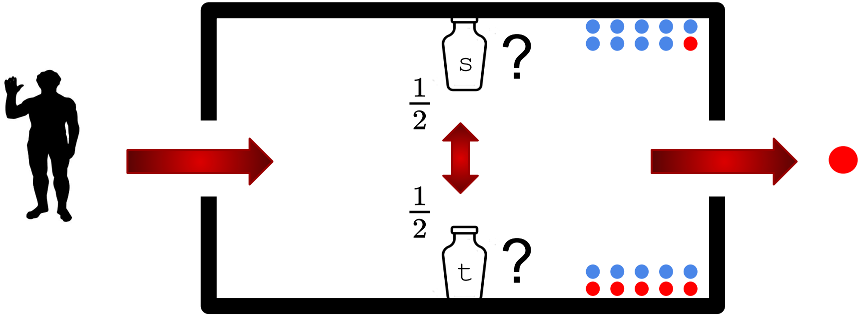


E = our player picked a red ball

T = our player picked the 't' urn

What is the probability of sampling a red ball given than I selected the urn 't'?

$$P(T|E) = \frac{\frac{1}{2} \quad \frac{5}{10}}{P(E)}$$

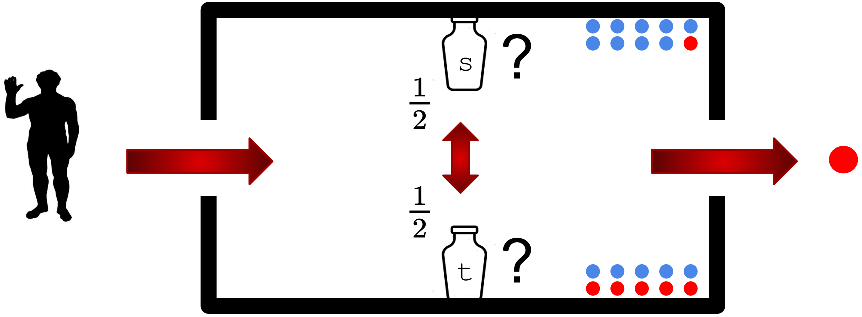


E = our player picked a red ball

T = our player picked the 't' urn

$$P(T|E) = \frac{\frac{1}{2} \quad \frac{5}{10}}{P(E)}$$

What is the probability of sampling a red ball altogether?

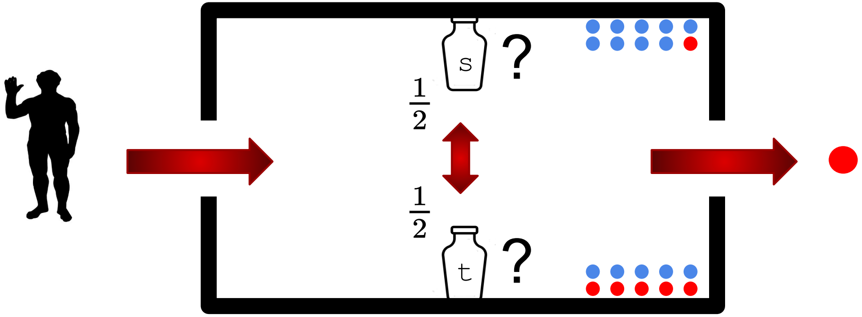


E = our player picked a red ball

T = our player picked the ‘t’ urn

$$P(T|E) = \frac{\frac{1}{2} \quad \frac{5}{10}}{\frac{6}{20}}$$

What is the probability of sampling a red ball altogether?



E = our player picked a red ball

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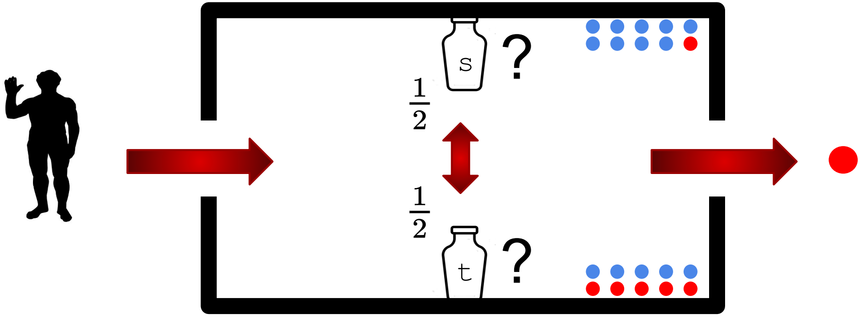
$$P(T|E) = \frac{\frac{5}{20}}{\frac{6}{20}}$$



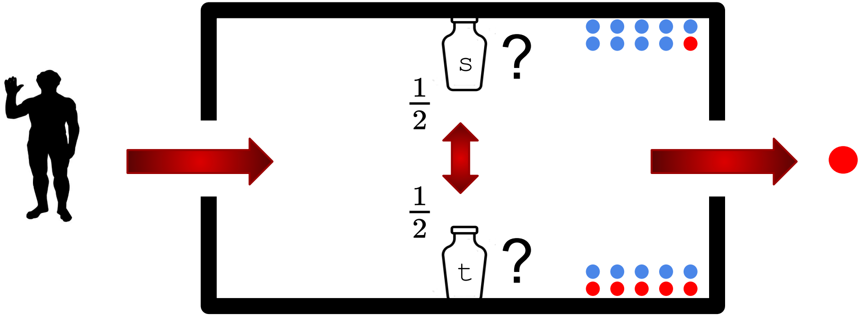
E = our player picked a red ball

T = our player picked the ‘t’ urn

Another way to visualize:

RED	RED	RED	RED	RED
RED	BLUE	BLUE	BLUE	BLUE
BLUE	BLUE	BLUE	BLUE	BLUE
BLUE	BLUE	BLUE	BLUE	BLUE

$$P(T|E) = \frac{\frac{5}{20}}{\frac{6}{20}}$$



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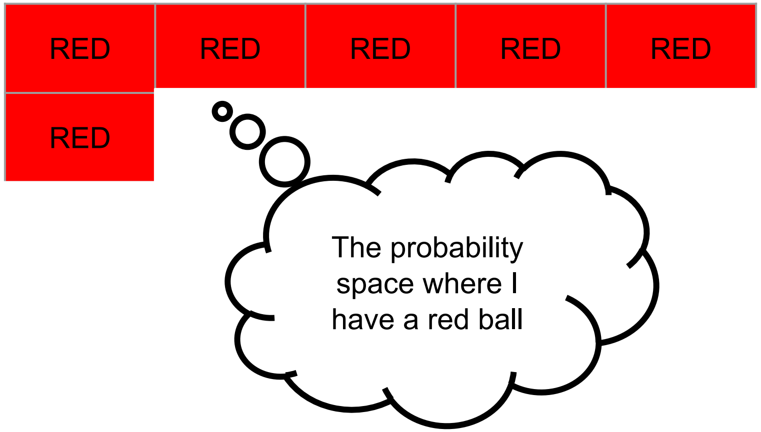

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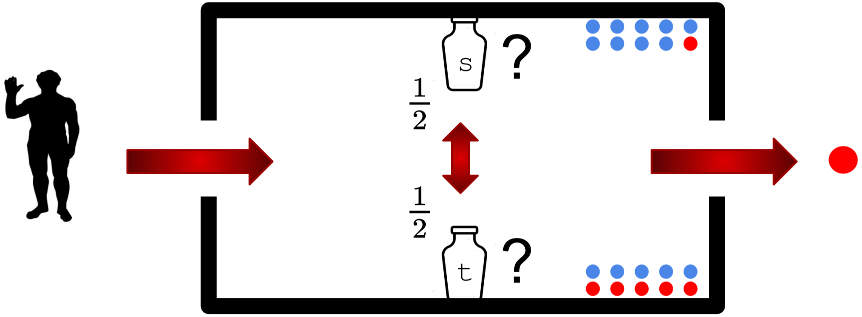
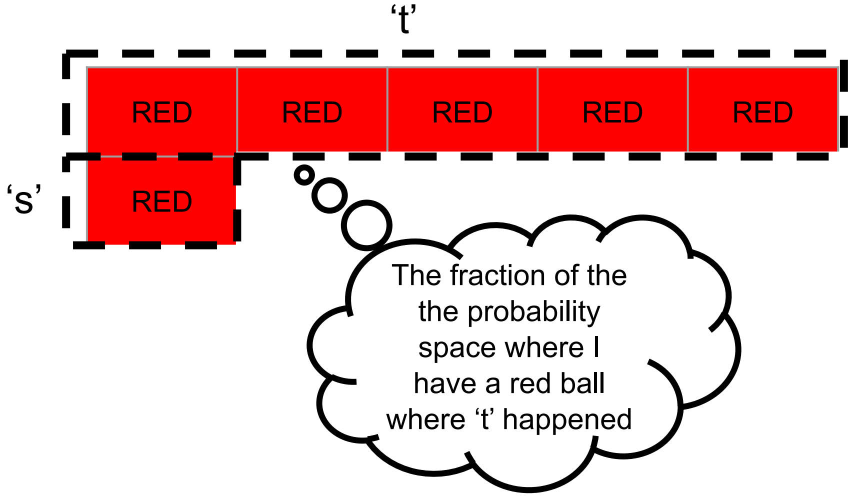
$$P(T|E) = \frac{\frac{5}{20}}{\frac{6}{20}}$$

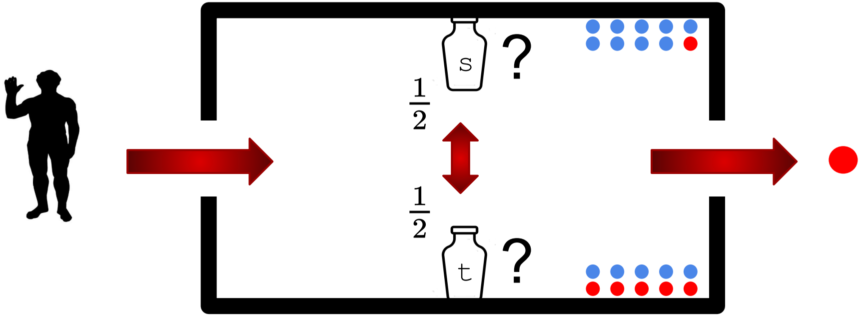




E = our player picked a red ball

T = our player picked the 't' urn

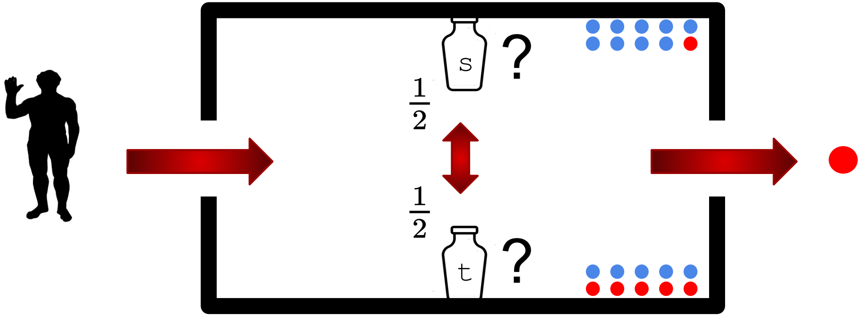
$$P(T|E) = \frac{\frac{5}{20}}{\frac{6}{20}}$$



E = our player picked a red ball

T = our player picked the 't' urn

$$P(T|E) = \frac{5}{6} \approx 83\%$$



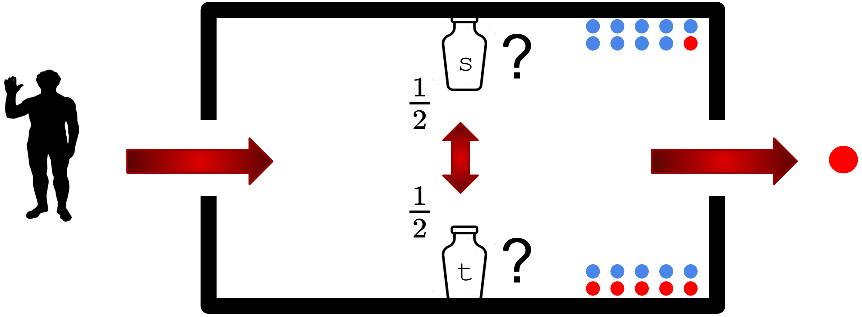
E = our player picked a red ball

T = our player picked the ‘t’ urn

Let us think about Bayes’ theorem a bit more...

- The color of the ball is an observation
- The urn that was selected is a piece of information I cannot have access to, a mental model
- I made a prediction about the probability of a model being correct given an observation

$$P(T|E) = \frac{P(T)P(E|T)}{P(E)}$$



M = a model

D = our data, our observation

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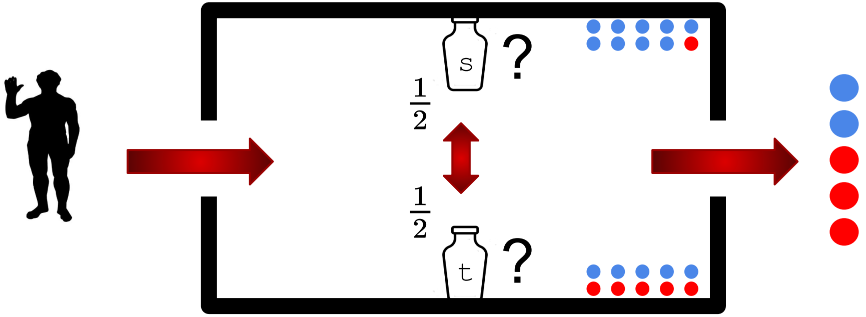
- I made a prediction about the probability of a model being correct given an observation

$$P(M|D) = \frac{P(M)P(D|M)}{P(D)}$$



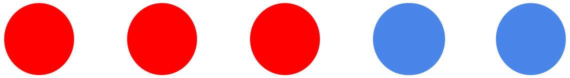
Let us think about Bayes' theorem a bit more...

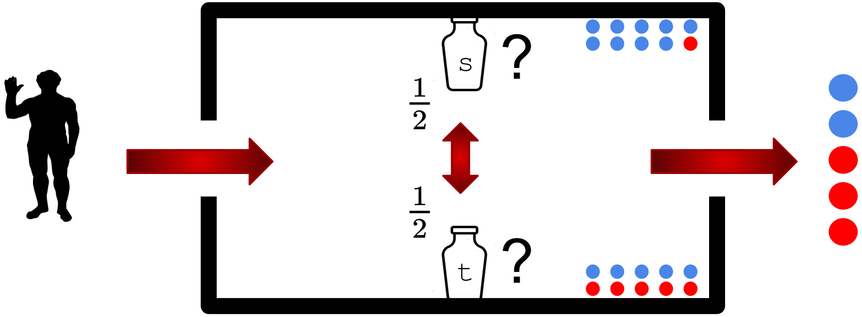
- Say our player:
  - selects an urn at random
  - picks a ball
  - records it
  - picks a ball again the **same** urn
- Our player does this 5 times
- When he leaves, he reports his observations

M = a model

D = our data, our observation

Observations 1:





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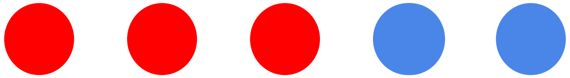
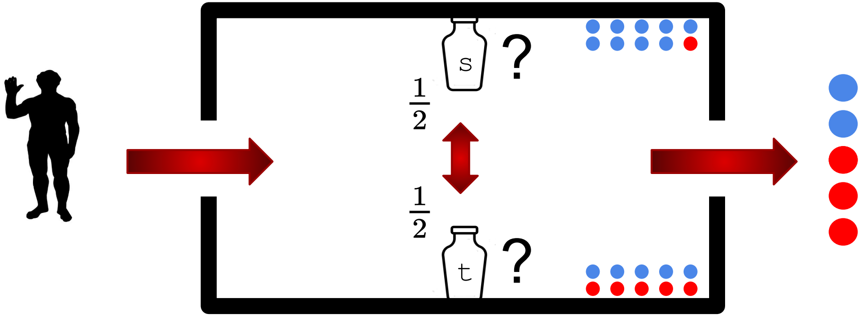
Observations 1: 

What is the probability that urn 't' was selected? ~97%



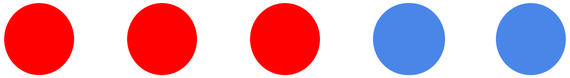
M = a model

D = our data, our observation

key ideas:

- Additional independent observations can give us more confidence in a model being the correct one
- Confidence is never absolute

Observations 1:



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